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Movement of a Particle Along an Inclined Cylinder Rotating Around Its Axis

Abstract. It is well known that parts of agricultural machinery often have a cylindrical shape. This shape, for example, can be observed in the casing of lifting and transport machines, where the active working body rotates. Furthermore, drum grain dryers and triers use an inclined cylinder that rotates around its axis. In this case, the particles of the technological material interact with the rotating surface, which leads to their sliding, the nature of which depends on the value of the angle of inclination of the cylinder. In this study, the methods of differential geometry, vector algebra, theoretical mechanics, and numerical integration of differential equations consider the motion of a particle along the inner surface of an inclined cylinder rotating at a constant angular velocity around its axis. The axes of a fixed coordinate system are used to compose differential equations of motion. It was established that the proper initial conditions under which the particle would be stationary at a certain distance from the lower forming cylinder towards its rotation can be determined analytically. In case of movement along an inclined cylinder, the particle moves, among other things, in the axial direction, while reducing the amplitude of vibrations. Furthermore, it was found that the angle of inclination of the cylinder plays a significant role. If the latter is less than the angle of friction, then the vibrations stop, the movement of the particle stabilises, and it performs a rectilinear movement at a constant speed in the axial direction. If the angle of inclination of the cylinder is greater than or equal to the angle of friction, then the particle moves rapidly in the axial direction and its movement does not stabilise. The value of the angular velocity of rotation also plays a significant role. A certain amount of it provokes "sticking" of the particle, which does not depend on the inclination angle of the cylinder. The obtained analytical dependences can be used in the design of cylindrical working bodies of agricultural machines

Keywords: surface, rotational motion, sliding, angular velocity, differential equations, trajectory

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INTRODUCTION

Cylindrical surfaces form an integral part of agricultural machinery. In lifting and transport machines, they play the role of a casing, inside which the active working body rotates (e.g., an auger). An inclined cylinder that rotates around its axis is used in drum grain dryers and cylindrical separators (triers). The interaction of material particles with the surface of the cylinder rotating around its axis leads to their sliding, the nature of which depends on the value of the inclination angle of the cylinder. An analytical description of the particle's sliding on the cylinder surface is a prerequisite for designing machines with such a nature of particle interaction with a moving surface, which indicates the relevance of this study.

Furthermore, at all times, the issues of increasing the durability of machines and their elements by improving reliability indicators have been extremely relevant. Thus, the article [1] describes the latest method of surface sulphidation developed. In [2], the method of surface hardening of parts using cementing and nitriding is highlighted. In [3], a method for applying a multi-layer coating is proposed. As the researchers note, such recommendations are insufficient to meet the needs of the present [4]. Notably, in all these and similar studies, the latest technologies of surface hardening are developed, which is a costly process.

Engineering practice often faces problems of geometric design of objects that are proposed to be solved in various ways, e.g., in multidimensional space by approximating the solution of differential equations [5], by multidimensional parabolic interpolation [6], by interpolating geometric space [7], etc. It is even easier to solve the inverse problem. In this case, the geometric design of technological objects is reduced to finding analytical dependencies of their interaction. Such interaction in mechanical engineering is the interaction between the working body

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and the material. This occurs during grain separation [8], aspiration separation [9], soil fertilisation [10], etc. The motion of an individual particle cannot be identified with the motion of a material that comprises individual particles, but it allows identifying patterns of motion that can be transferred to the material in a certain way. Thus, [11] presents the results of studies of particle motion on rough surfaces, and [12] – in rotary scatterers.

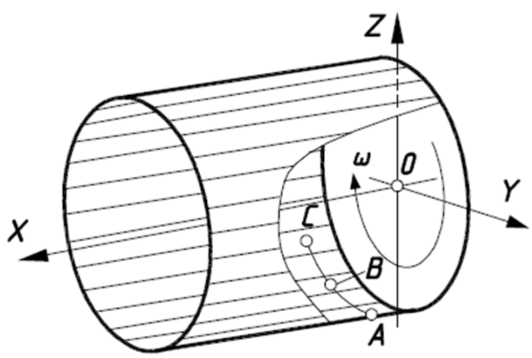
However, the study of body motion in some cases can be reduced to a particle [13]. This applies to the case when the inertial forces from the rotation of a body can be ignored due to the small angular velocities of their rotation [14]. Using this approach, the motion of particles on the surface of a spherical segment rotating around a vertical axis was investigated [15]. Thus, the range of applied problems requiring an analytical description of the motion of a particle along a plane is expansive. Proceeding from the above, the purpose of this study was to search for patterns of motion of a material particle along the inner surface of a cylinder set at an angle to the horizon and rotating around the axis. Research on this area is made in the monograph [16, p. 468], but they have a limited nature of calculations and graphical constructions for objective reasons due to the lack of modern computer tools at that time.

MATERIALS AND METHODS

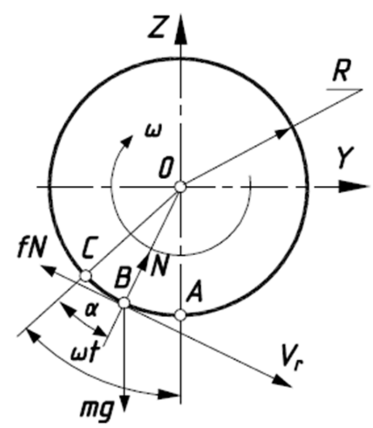
To fulfil the purpose set, the authors of this paper used methods of differential geometry, vector algebra, theoretical mechanics, and methods of numerical integration of differential equations. The parametric equations of a cylinder with a horizontal axis directed along the *OX* axis (Fig. 1) have the following form:

$$X = u; Y = R\,sin\,\alpha\,; Z = -R\,cos\,\alpha, \tag{1}$$

where R is the cylinder radius-constant value; α and u are the independent variable surfaces, and α is the angular coordinate, u is the linear coordinate (length of a rectilinear generating cylinder). The sign "–" in the above Equation (1) is taken so that the value α=0 corresponds to the lowest element of the cylinder, on which the particle will be in the initial position.



a)



b)

Figure 1. Graphical illustrations for compiling the equations of motion of a particle along the inner surface of a horizontal cylinder that rotates around its axis: a) axonometric view of the cylinder; b) projection of the cylinder, when the *OX* axis is directed at the observer and the forces applied to the particle at point *B*

At the beginning of the movement, the particle is at point *A* on the lower base of the cylinder (Fig. 1). The cylinder will be rotated around the axis with a constant angular velocity ω. During time t, the cylinder will turn to the angle ωt and the lower generator of the cylinder will move to point *C*. During the same time, the particle will move along the surface of the cylinder, but will not reach point *C*, as it will slide along it (Fig. 1b). Suppose it reached point *B*, which corresponds to angular slip α. Since the axis of rotation of the cylinder is horizontal, it is obvious that the sliding trajectory will be an arc of a circle. If the cylinder is tilted, there will be a component of the force of gravity, which will make the particle slide towards the *OX* axis as well. First, the study considered the movement of a particle on the surface of a horizontal cylinder.

If one connects the independent variables α and u of the surface by a certain functional dependence, e.g., on time t, then Equations (1) will turn into equations of one variable, i.e., they will describe a line on the surface of the cylinder. This line will be considered as the sliding trajectory, and the dependences α=α(t) and $u=u(t)$ are unknowns that need to be found. To find them, one needs to compile a system of differential equations of motion of the particle in projections on the axis of the *OXYZ* coordinate system.

The equation of motion of the particle will be formulated in the following form: $m\bar{w}=\bar{F}$, where m is the particle mass, $\bar{w}$ is the vector of absolute acceleration, $\bar{F}$ is the resulting vector of forces applied to the particle. These forces are weight force mg (g=9.81 m/s^2 – acceleration of free fall), surface reaction N, and friction force $f{\cdot}N$ (f – friction coefficient). Next, the direction cosines are found, i.e., the unit direction vectors of action of these forces. The weight force is directed downwards, so the projections of the guide vector will be written as follows:

$$mg: \{0; 0; -1\} \tag{2}$$

The frictional force $f{\cdot}N$ is directed opposite to the velocity vector of the relative movement V_r – sliding. To find the speed V_r of relative motion, Equation (1) is differentiated with respect to time t. Therewith, it is implied that α=α(t) and $u=u(t)$, i.e., Equations (1) are no longer equations of the cylinder, but equations of a line on it, i.e., of a relative trajectory. To distinguish between line and surface equations, in the equations of the relative trajectory, the authors switch from uppercase letters to lowercase letters with the index "r":

$$x_r' = u';\, y_r' = R\alpha' \cos\alpha\,;\, z_r' = R\alpha' \sin\alpha \tag{3}$$

The relative sliding velocity of a particle on the surface of a cylinder is defined as the geometric sum of the components (3):

$$V_r = \sqrt{x_r'^2 + y_r'^2 + z_r'^2} = \sqrt{u'^2 + R^2\alpha'^2} \tag{4}$$

When dividing (3) by (4), it is possible to obtain a unit vector of the tangent to the relative trajectory in the projections on the axis of the $OXYZ$ system. Given the opposite direction of the friction force $f{\cdot}N$ and the relative velocity vector V_r, the sign of the unit direction vector of the friction force must be changed to the opposite:

$$fN:\left\{-\frac{u'}{\sqrt{u'^2 + R^2\alpha'^2}};-\frac{R\alpha' \cos\alpha}{\sqrt{u'^2 + R^2\alpha'^2}};-\frac{R\alpha' \sin\alpha}{\sqrt{u'^2 + R^2\alpha'^2}}\right\} \tag{5}$$

Surface reaction N is directed from a point on the cylinder to the axis of rotation (Fig. 1b). If the radius vector of a point on a cylinder is determined by the second and third expressions of Equations (1), then the surface reaction is determined by the same expressions, but with the opposite sign. Having shortened the expressions by R, the projections of the unit reaction vector N will be written as follows:

$$N:\{0; -\sin\alpha\,; \cos\alpha\} \tag{6}$$

At the angular rotation speed ω of the cylinder surface during the time t, a turn by an angle of $\theta=-\omega t$ (clockwise) will be made. The generator of the cylinder, which was in the lower position at point A, will take the position at point C (Fig. 1b). To rotate the cylinder (1) around the OX axis by an angle $\theta=-\omega t$, the known formulas are applied as follows:

$$\begin{aligned} X &= u; \\ Y &= R\sin\alpha\cos\theta + R\cos\alpha\sin\theta\,; \\ Z &= R\sin\alpha\sin\theta - R\cos\alpha\cos\theta. \end{aligned} \tag{7}$$

After simplifications considering $\Theta=-\omega t$, equation (7) takes the following form:

$$\begin{aligned} X &= u; \\ Y &= -R\sin(\omega t - \alpha)\,; \\ Z &= -R\cos(\omega t - \alpha). \end{aligned} \tag{8}$$

Equation (8) at α=α(t) and $u=u(t)$ are the equations of the absolute trajectory of the particle's motion. The cylinder turned to the angle $\theta=-\omega{\cdot}t$, and during this time the particle, sliding along the cylinder in the opposite direction, turned to the angle α=α(t) and took the position at point B (Fig. 1b). The absolute speed of the particle is found by differentiating equations (8), switching to the lowercase letters with the index "a":

$$\begin{aligned} x_a' &= u'; \\ y_a' &= -R(\omega - \alpha')\cos(\omega t - \alpha)\,; \\ z_a' &= R(\omega - \alpha')\sin(\omega t - \alpha). \end{aligned} \tag{9}$$

By differentiating Equations (8), the projections of the absolute acceleration vector are found as follows:

$$\begin{aligned} x_a'' &= u''; \\ y_a'' &= R(\omega - \alpha')^2\sin(\omega t - \alpha) + R\alpha''\cos(\omega t - \alpha)\,; \\ z_a'' &= R(\omega - \alpha')^2\cos(\omega t - \alpha) - R\alpha''\sin(\omega t - \alpha). \end{aligned} \tag{10}$$

The unit direction vector of the action of the friction force $f{\cdot}N$ (5) and the reaction N of the surface (6) were found for a stationary cylinder. The surface rotates at an angle $\theta=-\omega{\cdot}t$, and therefore the vectors must also be rotated by this angle. Otherwise, the correspondence to the location of the particle is lost. The rotation is performed by analogy with the rotation of the surface according to Equations (7). After such manipulations, the projections of vectors take the following form:

– of the unit direction vector of the friction force $f{\cdot}N$:

$$fN:\left\{-\frac{u'}{\sqrt{u'^2 + R^2\alpha'^2}};-\frac{R\alpha' \cos(\omega t - \alpha)}{\sqrt{u'^2 + R^2\alpha'^2}};\frac{R\alpha' \sin(\omega t - \alpha)}{\sqrt{u'^2 + R^2\alpha'^2}}\right\}. \tag{11}$$

– of the unit direction vector of the action of the reaction force N:

$$N:\{0; \sin(\omega t - \alpha)\,; \cos(\omega t - \alpha)\} \tag{12}$$

Since the projections (10) of the absolute acceleration vector and the direction vectors of the applied forces of the particle weight mg (2), friction $f{\cdot}N$ (11) and reaction N (12) are known, the vector equation $m\bar{w}=\bar{F}$ must be formulated relative to the fixed $OXYZ$ coordinate system:

$$\begin{aligned} mx_a'' &= -fN\frac{u'}{\sqrt{u'^2 + R^2\alpha'^2}}; \\ my_a'' &= -fN\frac{R\alpha' \cos(\omega t - \alpha)}{\sqrt{u'^2 + R^2\alpha'^2}} + N\sin(\omega t - \alpha)\,; \\ mz_a'' &= -mg + fN\frac{R\alpha' \sin(\omega t - \alpha)}{\sqrt{u'^2 + R^2\alpha'^2}} + N\cos(\omega t - \alpha). \end{aligned} \tag{13}$$

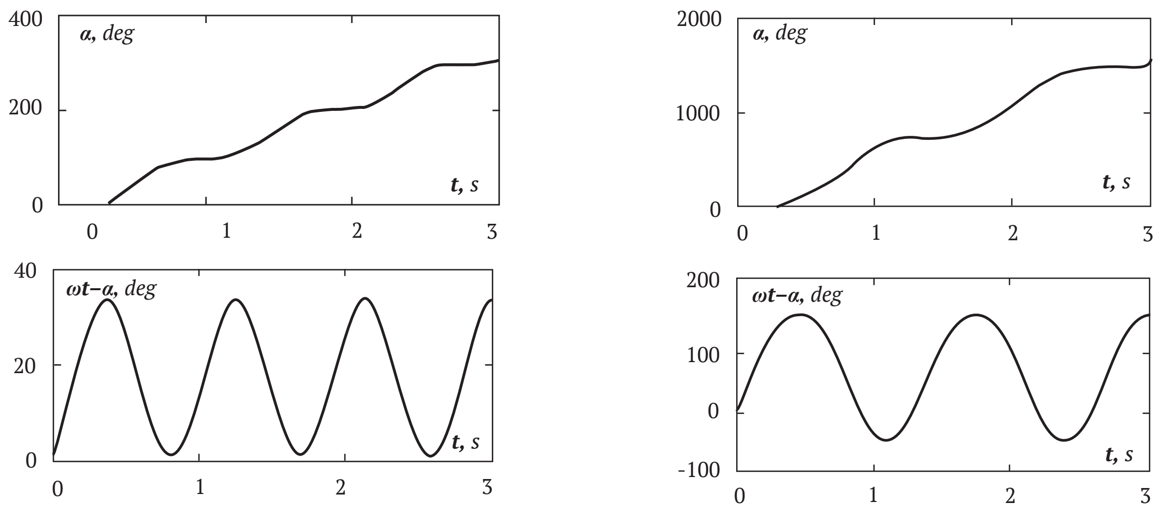
By substituting acceleration expressions (10) into (13), a system of three equations will be obtained with three unknown dependencies: α=α(t), $u=u(t)$, and $N=N(t)$. It should be used in the case when the initial velocity u' of particle sliding is specified towards the OX axis. When $u''=u'=0$ (i.e., when a particle slides around a circle), the first Equation (13) turns into the identity 0=0. Having solved the system of the other two with respect to α''=α''(t) and $N=N(t)$, one obtains as follows:

$$\alpha'' = \frac{g}{R}[\sin(\omega t - \alpha) - f\cos(\omega t - \alpha)] - f(\omega - \alpha')^2. \tag{14}$$

$$N = m[R(\omega - \alpha')^2 + g\cos(\omega t - \alpha)]. \tag{15}$$

Equation (14) is differential and can be solved independently. It can be assumed that when the horizontal cylinder rotates, the particle located at the bottom (point A, Fig. 1b) will rotate with the cylinder without sliding to point C, and then it will slide down to a certain point below, and this process will be repeated. Numerical integration of

Equation (14) has shown that this assumption is valid only for small angular velocities. An essential role in numerical integration is played by the initial conditions, on which, as will be shown later, the nature of the particle's motion depends. The initial conditions provided that the particle at the initial moment is on the bottom of the cylinder and there is no sliding angular velocity, i.e., α=α′=0. In Figure 2, graphs of the change in the kinematic characteristics of the particle movement during 3 s at *R*=0.2 m, *f*=0.3 and different angular velocities of the cylinder rotation are plotted. Above is a graph of the change in the angle α, and below – the difference in the angles ω·t–*α*. The horizontal section of the graph α=α(t), which is repeated periodically, indicates that at this moment in time there is no sliding, the particle "sticks" and rotates together with the cylinder. "Sticking" (lifting up) periodically alternates with sliding (lowering down). The graph of the change in the angle difference ω·t–*α* shows the amplitude of oscillations in the angular dimension.



a) **b)**

Figure 2. Graphs of changes in the sliding angle α and the angle of deviation of the particle ω·t–*α* from the zero value in absolute motion: a) ω=2 s^{-1}; b) ω=10 s^{-1}

The graphs show that the amplitude of particle vibrations increases as the angular velocity of rotation of the cylinder increases. If at ω=2 s^{-1} the particle turned by approximately 35° while rising and descended almost to zero (i.e., to the lower generator), then at ω=10 s^{-1} these angles are 165° and –40°, respectively, i.e., the particle oscillates in a circle, covering more than half of its arc. With the further growth of the angular velocity ω of the cylinder rotation, the particle practically "sticks" and rotates together with it.

If, at the initial moment, the particle is given an angular speed of sliding α′=ω, i.e., at the beginning of the movement, its absolute speed of rotation will be equal to zero, then the further movement of the particle will differ from the considered cases. For instance, the authors of this paper took ω=10 s^{-1} (the graph is presented in Figure 2b below for the initial conditions α=α′=0). Next, only one initial condition was changed: α=0, *α′*=*ω*. This substitution substantially changed the nature of the vibrations – their amplitude decreased (Fig. 3a).

a) **b)**

Figure 3. Graphs of changes in the particle deviation angle ω·t–*α* from the zero value in absolute motion at ω=10 s^{-1} and different initial conditions of integration: a) α′=ω, α=0; b) α′=ω, α=–15°

Theoretical studies have shown that when the angular speed of rotation of the cylinder increases, the particle does not "stick", but oscillates with the same amplitude (within 0° to 35°), i.e., the angular speed of rotation of the cylinder in this case does not affect the amplitude of oscillations. However, the second initial condition added to the first has an effect. In Figure 3b, a similar graph is plotted when α=–15°, i.e., at the initial moment, the particle is not fed to the lower point of the cylinder, but slightly higher towards its rotation. The amplitude of vibrations has decreased and is within 15°...18.5°, while it is not difficult to notice that in both cases (Fig. 3a, b) vibrations occur relative to the midpoint, which in angular measurement is approximately 17°. If α=–17° is taken as the starting condition, then the amplitude of oscillations practically disappears, and the particle remains motionless in absolute motion. This is confirmed by the partial solution of differential Equation (14) for the corresponding initial conditions.

Let the solution of equation (14) be the dependence $\alpha=\omega\cdot t+\alpha_o$. Then $\alpha'=\omega$, $\alpha''=0$. Substituting these expressions into the differential Equation (14) satisfies it. One gets a simple equation:

$$-\sin\alpha_0 - f\cos\alpha_0 = 0,\ \text{hence}\ \alpha_0 = -\mathrm{Arctg}f. \qquad (16)$$

Thus, the angle α_o is equal to the angular friction of the particle on the cylinder surface. For the accepted value of f=0.3, the angle α_o= –16.7°. If these initial conditions are met ($\alpha'=\omega$ and $\alpha=\alpha_o=-Arctg\,f$), the particle will slide along the surface of the cylinder, remaining motionless in absolute motion at a certain height from the lower generator, which in the angular dimension corresponds to the angular friction.

To make differential equations of motion of a particle along an inclined cylinder rotating around its axis, one needs to rotate the cylinder and bring all the vectors of forces and absolute acceleration according to its position. Rotation will be performed around the *OY* axis by an angle β, clockwise, so that the direction of the particle sliding down along the generating lines of the cylinder coincides with the direction of the *OX* axis. The weight force vector is directed downwards, i.e., it does not change its direction. The vectors of the remaining forces and absolute acceleration are rigidly tied to the surface of the cylinder or to the lines (trajectories) on it, so they must be rotated in the same way as the cylinder.

After turning by the angle β, the following expressions are obtained:

are the parametric equations of the cylinder:

$$\begin{aligned} X &= u\cos\beta - R\sin\beta\cos\alpha\,; \\ Y &= R\sin\alpha\,; \\ Z &= -u\sin\beta - R\cos\beta\cos\alpha\,; \end{aligned} \qquad (17)$$

are the projections of the absolute trajectory:

$$\begin{aligned} x_a &= u\cos\beta - R\sin\beta\cos(\omega t-\alpha)\,; \\ y_a &= -R\sin(\omega t-\alpha)\,; \\ z_a &= -u\sin\beta - R\cos\beta\cos(\omega t-\alpha)\,; \end{aligned} \qquad (18)$$

are the projections of absolute acceleration:

$$\begin{aligned} x''_{a\beta} &= R\sin\beta\,(\omega-\alpha')^2\cos(\omega t-\alpha) + u''\cos\beta - R\alpha''\sin\beta\sin(\omega t-\alpha)\,; \\ y''_{a\beta} &= R(\omega-\alpha')^2\sin(\omega t-\alpha) + R\alpha''\cos(\omega t-\alpha)\,; \\ z''_{a\beta} &= R\cos\beta\,(\omega-\alpha')^2\cos(\omega t-\alpha) - u''\sin\beta - R\alpha''\cos\beta\sin(\omega t-\alpha). \end{aligned} \qquad (19)$$

– of the unit direction vector of the friction force $f{\cdot}N$:

$$fN:\left\{\begin{aligned} &\frac{R\alpha'\sin\beta\sin(\omega t-\alpha) - u'\cos\beta}{\sqrt{u'^2+R^2\alpha'^2}}; \\ &-\frac{R\alpha'\cos(\omega t-\alpha)}{\sqrt{u'^2+R^2\alpha'^2}}; \\ &\frac{R\alpha'\cos\beta\sin(\omega t-\alpha) + u'\sin\beta}{\sqrt{u'^2+R^2\alpha'^2}} \end{aligned}\right\}. \qquad (20)$$

– of the unit direction vector of the action of the reaction force *N*:

$$\{N:\ \sin\beta\cos(\omega t-\alpha)\,;\sin(\omega t-\alpha)\,;\cos\beta\cos(\omega t-\alpha)\} \qquad (21)$$

Similarly, as for a horizontal cylinder, a system of differential equations is compiled considering the rotated vectors (19–21):

$$\begin{aligned} mx''_{a\beta} &= fN\frac{R\alpha'\sin\beta\sin(\omega t-\alpha) - u'\cos\beta}{\sqrt{u'^2+R^2\alpha'^2}} + N\sin\beta\cos(\omega t-\alpha)\,; \\ my''_{a\beta} &= -fN\frac{R\alpha'\cos(\omega t-\alpha)}{\sqrt{u'^2+R^2\alpha'^2}} + N\sin(\omega t-\alpha)\,; \\ mz''_{a\beta} &= -mg + fN\frac{R\alpha'\cos\beta\sin(\omega t-\alpha) + u'\sin\beta}{\sqrt{u'^2+R^2\alpha'^2}} + N\cos\beta\cos(\omega t-\alpha). \end{aligned} \qquad (22)$$

Next, the authors substituted the acceleration expressions (19) into (22) and solved with respect to $\alpha''=\alpha''(t)$, $u''=u''(t)$ and $N=N(t)$:

$$\begin{aligned} \alpha'' &= \frac{g}{R}\cos\beta\sin(\omega t-\alpha) - \frac{f\alpha'}{\sqrt{u'^2+R^2\alpha'^2}}[g\cos\beta\cos(\omega t-\alpha) + R(\omega-\alpha')^2]; \\ u'' &= g\sin\beta - \frac{fu'}{\sqrt{u'^2+R^2\alpha'^2}}[g\cos\beta\cos(\omega t-\alpha) + R(\omega-\alpha')^2]; \\ N &= m[g\cos\beta\cos(\omega t-\alpha) + R(\omega-\alpha')^2]. \end{aligned} \qquad (23)$$

RESULTS AND DISCUSSION

Using numerical integration, the kinematic characteristics of the movement of the particle along the inner surface inside the cylinder were obtained, and they substantially depend on two parameters: the angle of inclination β of the cylinder and the angular velocity ω of its rotation. In Figure 4, the absolute trajectories of the particle movement are plotted for a cylinder of radius R=0.2 m, tilted at an angle β=15° (smaller than the friction angle, which for f=0.3 is 16.7°).

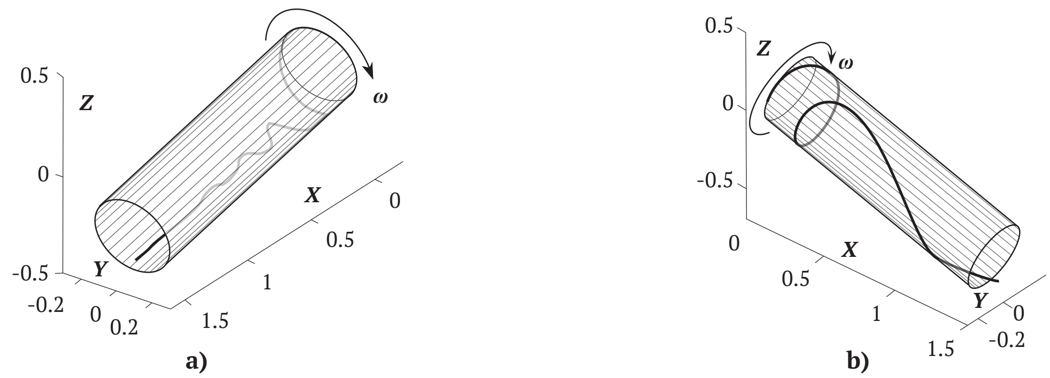


Figure 4. Absolute trajectories of particle motion along the inner surface of the cylinder at different angular velocities of its rotation: a) ω=2 s^{-1}; b) ω=10 s^{-1}

In Figure 5, graphs of speed changes are plotted $u'=V_z$ towards the OX axis. In one case (Fig. 5a), the tilt angle is smaller than the friction angle, in the second (Fig. 5b) – it is larger.

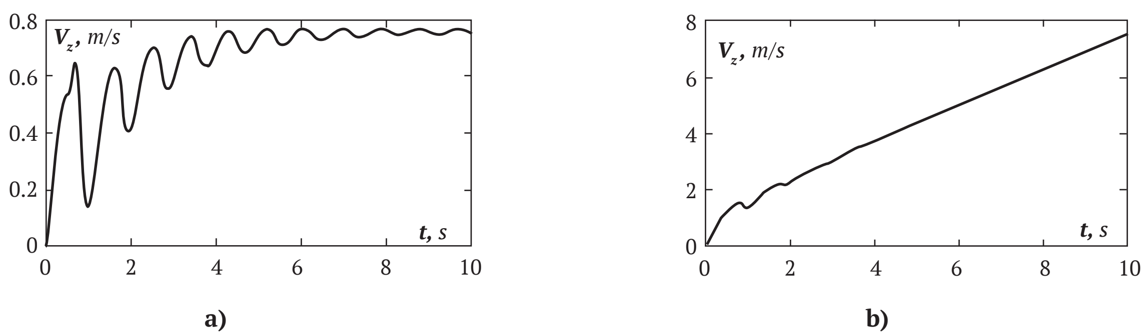


Figure 5. Graphs of the change in speed $u'=V_z$ towards the cylinder axis at ω=5 s^{-1} for different angles β of its inclination: a) β=10°; b) β=20°

Analysing the image in Figures 4a and 5a, it can be concluded that the motion of the particle stabilises, its velocity in the axial direction approaches a constant value, and the trajectory approaches a straight line. Such stabilisation is possible up to a certain value of the angular velocity of the cylinder rotation. Figure 4b shows the absolute trajectory at ω=10 s^{-1}, which shows that the particle's oscillations are increasing. As the angular velocity ω increases further, the particle practically "sticks" and rotates together with the cylinder. At low angular speeds of rotation of the cylinder (i.e., before the particle "sticks"), speed stabilisation is possible only for cylinder tilt angles that are less than the angle of friction. Figure 5b shows a graph of the particle sliding speed in the axial direction at an angle β greater than the friction angle. It shows that the sliding velocity of the particle increases linearly.

"Sticking" of the particle occurs when the corresponding angular velocity of rotation of the cylinder is reached at any angles of its inclination. Figure 6 shows graphs of the change in the sliding angle α and the distance u of the particle movement in the axial direction at ω=20 s^{-1} and the angle of inclination β=45°.

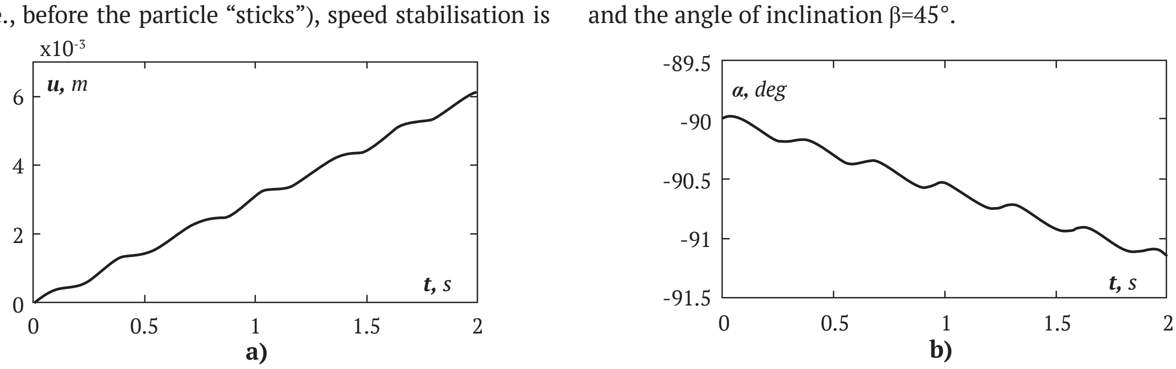


Figure 6. Graphs of changes in the kinematic characteristics of particle motion at ω=20 s^{-1} and the inclination angle β=45°: a) graph of the dependence $u=u(t)$; b) dependence graph α=α(t)

The graphs show that the particle moves 6 mm in the axial direction during the time t=2 s and turns 1° in the angular dimension. It is clear that a further increase in the angular velocity of rotation of the cylinder will lead to complete "sticking" of the particle.

The results obtained considerably complement the similar results obtained in the monograph [16]. In the given study, the sliding trajectories of particles are plotted on the surface of an inclined cylinder, in contrast to the specified monograph, where they are plotted on a cylinder sweep. Furthermore, this study considered cases where the angle of inclination of the cylinder is less than or greater than the angle of friction, while the aforementioned study considered an angle less than the angle of friction. The results obtained show that with an increase in the angular velocity of rotation of the cylinder, "sticking" of the particle is possible, which was not considered in the cited monograph.

As it turned out, for the cylinder inclination angles smaller than the friction angle, it is possible to stabilise the motion, in which the particle slides in a straight line at a constant speed. A solution for this case and for a horizontal cylinder can be found by looking for the solution in the form $\alpha=\omega\cdot t+\alpha_o$, supplementing it with a constant velocity in the axial direction $u'=V_z=const.$ Thus, $\alpha'=\omega$, $\alpha''=0$, $u''=0$. Substituting these data into the first two Equations (23), a system of two equations is obtained as follows:

$$0=\frac{g}{R}\cos\beta\,\sin(-\alpha_0)-\frac{f\omega g\cos\beta\cos(-\alpha_0)}{\sqrt{V_z^2+R^2\omega^2}};\qquad 0=g\sin\beta-\frac{fV_z g\cos\beta\cos(-\alpha_0)}{\sqrt{V_z^2+R^2\omega^2}}.\quad(24)$$

After solving system (24) with respect to α_o and V_z, one obtains:

$$\alpha_0=-\mathrm{Arctg}\sqrt{f^2\cos^2\beta-\sin\beta};\,V_z=R\omega\sqrt{\frac{1+f^2}{f^2\mathrm{ctg}^2\beta-1}}.\quad(25)$$

The obtained result (25) should be understood as follows: if a particle hits a cylinder at α_o with a relative angular velocity equal to the angular velocity of rotation of the cylinder, directed oppositely from the direction of its rotation and with a relative translational velocity along the V_z axis, then it continues to move at this speed along the cylinder without oscillation.

CONCLUSIONS

When a particle hits the inner surface of a horizontal cylinder, which rotates with an angular velocity ω around its axis, it begins to oscillate in the plane of the cylinder cross-section with a certain amplitude in the angular dimension. The magnitude of the amplitude depends on the point of impact of the particle, the coefficient of friction, and the initial absolute velocity. Under proper starting conditions, which are determined analytically, a particle in absolute motion can be stationary, being at a point in the cylinder at a certain distance from the lowest point in the angular dimension along the course of the cylinder rotation. When the cylinder is tilted at an angle β to the horizon, the particle begins to move in the axial direction, while the amplitude of vibrations decreases. The angle of inclination of the cylinder is important: at the angle β, which is smaller than the friction angle, the movement stabilises, the oscillations stop, and the particle moves in a straight line in the axial direction with a constant speed; at an angle β greater than or equal to the angle of friction, motion stabilisation does not occur, the particle moves accelerated in the axial direction. The value of the angular velocity of rotation is of great importance. When a certain value is reached, the particle practically "sticks" regardless of the angle of inclination of the cylinder. Experimental verification of data obtained as a result of theoretical research can serve as the basis of further research.

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Переміщення частинки по похилому циліндру, що обертається навколо власної осі

Анотація. Загальновідомо, що частини сільськогосподарських машин часто мають циліндричну форму. Таку форму, наприклад, має кожух підйомно-транспортних машин, в якому обертається активний робочий орган. Крім того, у барабанних зерносушарках та трієрах використовується похилий циліндр, що обертається навколо власної осі. При цьому частинки технологічного матеріалу взаємодіють з обертовою поверхнею, що призводить до їх ковзання, характер якого залежить від величини кута нахилу циліндра. У даному дослідженні методами диференціальної геометрії, векторної алгебри, теоретичної механіки та чисельного інтегрування диференціальних рівнянь розглядається рух частинки по внутрішній поверхні похилого циліндра, що обертається з постійною кутовою швидкістю навколо власної осі. Для складання диференціальних рівнянь руху використано осі нерухомої системи координат. Встановлено, що належні вихідні умови, при яких частинка буде нерухомою на певній відстані від нижньої твірної циліндра в напрямі його обертання, можуть бути визначені аналітично. У випадку руху по похилому циліндру частинка рухається в тому числі в осьовому напрямі з одночасним зменшенням амплітуди коливань. Крім того, з'ясовано, що значну роль має кут нахилу циліндра. Якщо останній менше за кут тертя, то коливання припиняються, рух частинки стабілізується та вона здійснює прямолінійний рух зі сталою швидкістю в осьовому напрямі. Якщо кут нахилу циліндра більше або дорівнює куту тертя, то частинка рухається прискорено в осьовому напрямі та її рух не стабілізується. Також значну роль має величина кутової швидкості обертання. Певна її величина провокує «залипання» частинки, яке не залежить від кута нахилу циліндра. Отримані аналітичні залежності можуть бути використані при проектуванні циліндричних робочих органів сільськогосподарських машин

Ключові слова: поверхня, обертальний рух, ковзання, кутова швидкість, диференціальні рівняння, траєкторія