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PECULIARITIES OF TEACHING STUDENTS OF NON-MATHEMATICAL SPECIALTIES HIGHER MATHEMATICS IN CRISIS CONDITIONS

Abstract. The problem of students' mathematical training is relevant for many countries of the world. Scientists note a general tendency of decreasing the level of students' mathematical knowledge, the presence of significant difficulties in studying mathematical courses. The relevance of this problem intensifies when society experiences certain crisis phenomena. Countries of the world are living in conditions of pandemics, natural and ecological disasters, military operations, etc. We call such living conditions to be crisis ones. In such conditions students experience psychological stress, negative emotions, increases their feeling of anxiety, which complicates the organization of the educational process and reduces the quality of knowledge acquisition.

The article experimentally substantiates the conclusion that the quality of acquisition of higher mathematics' learning material by students of non-mathematical specialties improves significantly in case of using elements of "programmed learning" technology (in terms of content), "flipped classroom" technology (in terms of form) and visualization technology of educational material in crisis conditions. The authors provide methodical recommendations for the implementation of the technologies mentioned above. A conclusion about positive influence of the described methodical approach on the quality of students' assimilation of mathematical knowledge both in crisis and normal conditions of life and study was made.

Keywords: Higher mathematics, Educational technologies, Crisis conditions

Formulation of the problem. Modern society needs highly trained specialists. Ukraine, like all other industrialized countries, has a technical infrastructure that must be constantly maintained, developed and updated. Insufficient personnel training can lead to the decline of production, the destruction of infrastructure and the halting of civilizational development, which can have numerous negative consequences. Mathematical training occupies a special place in the training of future specialists who ensure the viability of the state. Recently the decline in the quality of mathematical training of specialists was noted in various areas. The problem is important and relevant. Understanding the relevance of this problem at the state level led to declaring the academic year 2020-2021 to be the year of mathematics education in Ukraine [1]. The problem of mathematical training of pupils and students is relevant for many countries of the world. Scientists note the general trend of decreasing the level of mathematical knowledge of pupils and students, presence of significant difficulties in studying mathematics courses [24, 29, 44], which affects the decrease in the level of their educational motivation or complete refusal from studying.

At the current stage of society's development, schools and universities use various methodical systems of teaching mathematics, which are theoretically grounded and experimentally verified. A school teacher and a university teacher can choose different forms and methods of teaching. Unfortunately, unexpected situations can suddenly occur and radically change everyday life. It happened in 2019, when many countries found themselves in the conditions of the Covid-19 pandemic. Military operations are underway in our country from February 24, 2022. Other countries suffer from natural and environmental disasters (earthquakes, hurricanes, floods, etc.). We call such conditions to be crisis ones. Any country in the world can find itself in such conditions.

It is obvious that in such conditions both students and teachers experience psychological stress, a very high level of anxiety. Negative emotions often lead to rejection from the “normal life”, the level of educational motivation decreases to an absolute unwillingness to learn. From the ethics point of view in the article we do not describe personal opinions of students and teachers in our university, their psychological state of mind, what they feel after the start of military operations on the territory of our state. We want to only mention that psychological support services work intensively at the university.

Our experience of teaching university students a higher mathematics course allowed us to conclude that in crisis conditions it is necessary to find all opportunities to continue the educational process, because it contributes to the stabilization of the psychological state of mind of university students and teachers. Additional efforts of teachers aimed at supporting educational motivation and positive emotional state of students are needed. The educational process in such conditions is possible only in the form of distance or blended learning.

As a result of the search for educational technologies that are effective in the conditions of distance or blended learning, we came to the assumption that in the process of teaching a higher mathematics course to students of non-mathematical specialties it is effective to use elements of the "programmed learning" technology (in terms of content) and elements of technology called "flipped classroom" (in terms of the form); the effectiveness of students' learning can be significantly increased in case of mandatory generalization and systematization of educational material in mathematics with the use of visualization tools.

In order to verify the effectiveness of the use of the mentioned technologies in the process of teaching higher mathematics to students of non-mathematical specialties, we conducted an experimental study, which involved the following tasks: analysis of theoretical provisions and experience of introducing programmed learning technology into the educational process, technology of the "flipped classroom", and technology of visualizing educational material; development of methodological recommendations for the implementation of the mentioned technologies in the process of teaching a higher mathematics course to students of non-mathematical specialties in accordance with the curriculum; experimental verification of the influence of the developed methodical recommendations on the quality of students' assimilation of mathematical knowledge.

Analysis of latest research and publications. The idea of programmed learning arose quite a long time ago. Modern researchers associate it with the works of the famous American psychologist S. Pressey, who patented special machines for testing students' knowledge in the 20s of the 20th century. The author of the developments believed that the assimilation of knowledge is going step by step, the transition to learning new material is possible only if correct answers are given to previous questions or the teacher corrects incorrect answers [31, 32]. The idea of programmed learning received further development in the works of the American psychologist B. Skinner. The scientist claimed that it is important to constantly «reinforce» the student with positive evaluations, stimulation in the learning process, that means encouraging the student with the help of achieving certain success every day in the so-called «short term» [38, 39]. Therefore, B. Skinner suggested dividing the educational material into small portions, which must be learned gradually. The idea of programmed learning was developed into a coherent theory in the works of the scientist. This theory was further developed in the works of various scientists, in the studies of psychologists and teachers of the Soviet educational system [7, 17, 20, 21, 26, 33]. The analysis of scientific studies on the substantiation of the theoretical foundations of programmed learning and its implementation in the schools' practice of this period allow us to draw a conclusion about the effectiveness of this form of education [15].

Today scientists continue the discussion about the advantages and disadvantages of distance and blended learning, the search for effective methods of distance learning of mathematics continues [14, 18, 28, 41, 43]. The technology of the «flipped classroom» is the most popular one. It can be interpreted as a «pedagogical model in which the typical elements of a lecture and homework are swapped. The flipped classroom approach relies on concepts such as active learning, … . The value of the flipped classroom is in changing time of a lecture into time for a workshop where students can question lecture content, test their application skills, and interact during hands-on sessions» [6]. A large number of articles are devoted to the comparative analysis of the effectiveness of traditional and flipped learning in distance and blended learning in universities [2, 9, 11, 12, 23, 24, 37]. Most authors confirm the conclusion that the technology of the «flipped classroom» is more effective compared to the traditional one in terms of students' success, their social interaction and educational motivation.

A specific feature of mathematical concepts is a high level of abstraction. This fact is the cause of difficulties in learning the educational material in mathematics for many students of various specialties. One of the effective means of teaching mathematics, which allows you to improve the understanding of mathematical facts, is the visualization of educational material. There are different approaches to the interpretation of the concept of «visualization». Researchers are looking at different aspects of this process. For example, visualization as a way of exploring the world [8]; visualization as a learning tool [25]. The importance of visualizing educational material in the process of learning mathematics has been substantiated rather long time ago. The analysis of the use of visualization tools in learning and teaching mathematics [3, 30] showed that the relevance of this problem is increasing in recent years.

Research into the role of visualization in mathematics learning and teaching continues. Scientists investigate the influence of visual representations of pupils and students on the ability to solve problems [13, 42]; the use of visualization techniques to increase the level of creativity of students [5]; the role of visual images in the process of solving problems from the theory of probabilities [45]; the use of various visualization tools in the earning process [4, 22]. The new stage of visualization research in education is associated with the development of information, communication, and digital technologies [16, 19, 35, 36, 40].

The purpose of this article. To substantiate the effectiveness the use of elements of "programmed learning" technology (in terms of content), elements of "flipped classroom" technology (in terms of form) and technology of visualizing educational material in the process of teaching a higher mathematics course to students of non-mathematical specialties in crisis conditions.

Presentation of the main research material. At the current stage of the pedagogical science, development programmed learning is understood as a system of theoretical provisions, organizational forms and means of educational work, which foresees mainly indirect program management of the cognitive activity of pupils (students) and specify their independent assimilation of knowledge, abilities, skills, and mental development of the individual. The term «programming» in relation to the educational process means the creation of programs that manage the educational activities of pupils (students) during solving cognitive tasks. Such programs are called machine learning (learning algorithms) [27].

Methodologists define the following features of training programs that are the basis of programmed training:

1) selection of educational material, its arrangement in a clear logical sequence;
2) elimination of non-essential information, dividing the material into certain parts;
3) precise and specific instructions for dealing with the tasks that are necessary for learning each portion of the material;
4) tasks for self-assessment (internal feedback), as well as for teacher's control (external feedback) during the process of students' assimilation of knowledge, skills, and abilities;
5) controlled move from one portion of educational material to another, the next one.

In practice we relied on the conclusion of scientists that the main, essential characteristics of programmed learning are: presentation of educational material in separate portions; constant two-way communication between the learner and the teacher.

The implementation of programmed training can take place according to the following scheme:

1) the teacher presents the first portion of the educational material, explains it, asks control questions; if the answers are correct, reports a new portion of educational material;
2) students perceive the first portion of the material accordingly, learn the content, answer the questions and move on to learning the next one;
3) after students have mastered the entire amount of material on this topic, the teacher asks questions that allow to summarize and systematize the educational material.

Students can receive educational information directly from the teacher or use programmed manuals. The material in such a manual can be presented in linear or branched systems. According to the linear system of material presentation, it is assumed that the educational information is divided into small portions that are not difficult to learn. Each portion is designed for the active response of the student, contains hints or instructions that make the search for the correct answer easier. Every answer should be checked on its correctness by the student. If the answer is

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incorrect, he continues to look for the correct one, and then moves on to another portion of educational information. A linear program eliminates mistakes and brings satisfaction from success. All students learn the same educational material, but in a different tempo. In textbooks built based on a branched system of material presentation, information is also divided into portions. The difference between a branched system and a linear system is that it has several answers to each question, and after learning a portion of information, you need to choose the right answer. If the answer is correct, the student moves on to learning the next portion of material, if it is incorrect, he additionally repeats the learned information. Such a textbook does not allow the student to progress further until he completes the task correctly. The focus is not on avoiding errors, but on their clarification and analysis [10]. The technology of programmed learning arose a long time ago and is not in scope of interest of modern textbook authors. We did not find special «programmed» mathematics textbooks. Therefore, we used our own developments in our work, which were built taking into consideration the principles of programmed learning. For example, students were offered structured lectures in which separate portions of the educational material were clearly identified. The specific features of the educational process organization during military operations are the unplanned interruption of classes, dividing them into parts (the need to respond to an alarm signal, turning off the lights, etc.). The learning process becomes «discrete», often occurring in small intervals. That is why features of programmed training like systematic selection of educational material, placing it in a clear logical sequence; elimination of non-essential information, division of material into certain parts; precise and specific instructions for performing the tasks necessary to learn each portion of the material; tasks for self-assessment, as well as for teacher's control; a controlled transition from one portion of educational material to another, the next one, allows to implement it in the distance form of education in the conditions of martial law. We claim that the use of elements of programmed learning allows to increase the effectiveness of distance learning of mathematics students in the conditions of martial law.

Lectures and practical classes are traditional in the process of learning mathematics. We singled out the following methodological features of conducting a lecture session taking into consideration the elements of programmed learning:

1. The topic and the main questions of the topic are clearly formulated.
2. Educational material is provided in small portions.
3. Tasks for learning each portion of the theoretical material are formulated (make an outline, write definitions, write down the formulation of theorems, make a corresponding drawing, make a general diagram, or fill in a table, etc.).
4. Students should send reports with the completed task to the teacher for verification (the deadline for submitting the reports is clearly determined by the teacher). No assessment is given after task check.

We would like to note that the content of the lecture can be presented sequentially or with a certain branching. Learning the theoretical material should be supported by examples of tasks with complete solutions and comments. At the practical lesson it is planned to solve tasks of the main types on the corresponding topic. Students are offered tasks of different levels of difficulty (the student himself chooses the level of task's difficulty) for self-completion. Students have the opportunity to solve problems using samples and comments. Solutions to tasks must be commented.

In the process of organizing mathematics education for students of the Sumy National Agrarian University during the last academic year, which took place under the conditions of martial law in Ukraine, we used elements of the «flipped classroom» technology. We need to mention that this technology was implemented not in a complete scheme, but partially. Let us comment on this fact. We think that the number of hours planned for the mathematics course at our university is not enough. But the content of the course is very informative. At the Sumy National Agrarian University, in accordance with the curriculum of the mathematics course, it is traditionally planned to get students acquainted with the elements of linear algebra, vector algebra, analytical geometry, differential and integral calculus, probability theory and mathematical statistics. The named topics contain a large amount of educational material. According to the curriculum, according to the number of classroom hours, we plan one lecture and two practical classes for studying each topic of the mathematics course at the university. We use the following scheme for organizing students' educational activities: practical lesson No. 1 - lecture - practical lesson No. 2.

The mathematics course studied by the students at our university is based on the school mathematics course and is a continuation of this course. We made a comparative analysis of the content of the main topics of the mathematics course studied by students at the university and the content of the school mathematics course. As a result of the analysis, we found out that the educational material of the mathematics course at the university contains approximately 30% to 50% of the basic concepts of school mathematics. We took this fact into account when organizing the educational activities of students, which consists of three main stages. Let us reveal the content of each stage.

First stage - practical lesson No. 1. At the first stage, the «flipped classroom» technology is implemented. The teacher gets the students acquainted in advance with the topic and the main questions that will be considered in the first practical session. These questions correspond to the content of the educational material of school mathematics. The teacher formulates tasks for students who independently process the educational material based on school textbooks (tasks are sent to students by e-mail) or other available sources of information. The experience of teaching mathematics under martial law allows us to state that it is advisable to clearly formulate the questions and indicate the source from which the answers should be found when students independently process the educational material of the school mathematics course,

In our work, we use the author's study guide [34], in which the school mathematics course is systematized according to the main content lines (appropriate educational material is sent to the student's e-mail together with the tasks, which is sufficient for completing the tasks). In the first practical lesson the educational material that the students have learned on their own is discussed.

The second stage – a lecture. The teacher reviews the main concepts of the topic, which are already known to the students, and introduces new concepts, using the techniques of analogy and comparison during the lecture. He focuses attention on the interrelationships of the studied concepts, using methods of generalization and systematization of educational material.

The third stage – practical lesson No. 2. Students do exercises of the main types and solve tasks to generalize and systematize the educational material on this topic (develop generalizing schemes and systematize tables, the structure of which is proposed by the teacher) in the second practical session.

Under normal conditions, before the military operations on the territory of our country, we offered students creative tasks, the performance of which required a certain level of research skills. Students performed group projects, the results of which were presented at the end of studying the topic. The control papers had individual versions of the tasks. In crisis conditions, according to our observations, most students have no (or insufficient) motivation to perform such tasks. Therefore, in order to improve the quality of assimilation of mathematical knowledge in crisis conditions, it is advisable to clearly formulate tasks, consider tasks of the main types and provide examples of solving such tasks. We even offered one version of the test tasks, and after announcing the grades for performance to the students, we made a general analysis of errors providing correct answers (it is impossible to do such a general analysis performing individual tests).

In our opinion, the use of special methods of visualization of educational material is effective at the stage of generalization and systematization of knowledge. In the process of teaching mathematics in crisis conditions, we consider the stage of generalization and systematization of educational material to be mandatory. The generalization of knowledge is considered as a method of mental and educational activity in didactics. The didactic content of the generalization means the selection of essential features, characteristics, formation and formulation of concepts, laws, ideas of the educational material. The result of the generalization of a certain level of knowledge is their systematization. That is why usually in didactics these two processes are connected and considered together. Generalization and systematization of knowledge are effective means of deepening, universalizing, organizing, understanding, and memorizing knowledge. Generalization of knowledge allows to develop the ability to solve problems by transferring the method of action to a whole class of similar problems, which is one of the main tasks of teaching mathematics.

Our own experience in teaching mathematics has shown that at the stage of generalization and systematization, two means of visualizing the educational material are the most effective:

1) tables, which are used to systematize the learning material of the topic (concepts, their properties, basic facts, formulas and equations), the section, as well as the entire mathematics course;

2) schemes that help to make connections between concepts, demonstrate the sequence of studying different questions of each topic, and different topics of the entire mathematics course.

Throughout the whole school year, we have been working in crisis conditions and made sure that in crisis conditions the best result is achieved if the basis of the tables and diagrams is offered by the teacher (in lectures or as a task for independent work). We believe that the basis of such visualization tools should be the same for all students, the content and structure should be clearly defined. This contributes to students' awareness of the logical relationships between the main facts of the educational material and allows to avoid possible mistakes in the process of their assimilation and systematization. Students are given the task to independently fill in the gaps, separate logical blocks, establish connections between concepts and show them with arrows. During the practical lesson, there is a discussion, students comment on the completed tables and diagrams, the teacher can demonstrate his version of the tables and diagrams. Students' independent processing of the means of sign-symbol visualization with their further discussion contributes to the awareness of the main concepts of each topic, logical connections between concepts and the main content parts of the educational material.

In our opinion, students should not be required to clearly formulate definitions and properties, to know formulas. We emphasize that the student should demonstrate the ability to apply a known fact to the solution of a certain type of problem (at the same time students can use diagrams, tables, lecture notes, as well as notebooks for practical classes) doing control tasks.

In order to check the effectiveness of the proposed methodological recommendations the experimental study was leaded by us. The experiment was conducted in the 2022-2023 academic year on the basis of the Sumy National Agrarian University. All specialties of the university can be called “non-mathematical”, so the mathematics program is small in scope; the mathematics curriculum contains only those topics that are necessary for professional activity or for mastering professional disciplines during further education.

71 students of the Faculty of Agricultural Technologies and 84 students of the Faculty of Economics took part in the experiment. Students of these specialties study higher mathematics in the first year for two semesters under one curriculum. A final

control of students' knowledge is conducted at the end of each semester. The university uses a 100-point universal scale as well as a national rating scale to assess students' knowledge. We distinguished four levels of students' mathematical knowledge according to the number of points they received in the course while studying higher mathematics to simplify calculations. Table 2 shows the correspondence between the number of points a student receives in the final control process and the students' levels of mathematical knowledge (Table 1).

Table 1.

Correspondence of the number of final control points received by a student, assessment on the ECTS scale, and the level of students' knowledge.

Final control points, assessment on the ECTS scale	0–34, F	35–59, FX	60–68, E	69–74, D	75–81, C	82–89, B	90-100, A
Level of knowledge	Elementary		Intermediate		Sufficient		High

In the course of the study the students of the Faculty of Agricultural Technologies were defined as the experimental group (EG), and the students of the Faculty of Economics were defined as the control group (CG). At the beginning of the experiment we made a comparative analysis of the success in higher mathematics of students of both groups. The results of the final control for the first semester (December 2022) were used for it. The distribution of students by levels of mathematical knowledge acquisition according to the final control results in higher mathematics of students of the experimental (EG) and control (CG) groups at the beginning of the experiment is presented in Figure 1.

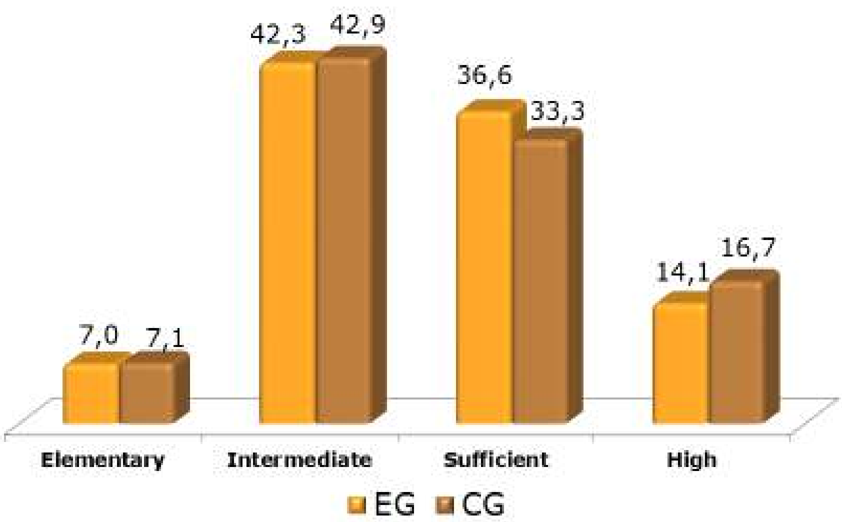

Fig.1. Students' distribution by levels of mathematical knowledge acquisition according to the final control results in higher mathematics of the experimental (EG) and control (CG) groups at the beginning of the experiment

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In order to establish the homogeneity of the experimental and control groups in terms of the success in studying higher mathematics course we used the method of testing statistical hypotheses. The null hypothesis is that in terms of success in higher mathematics, the control and experimental groups are the same (at the chosen significance level α=0.05). The alternative hypothesis is that students of the control and experimental groups differ significantly according to this characteristic. Hypotheses were tested using the χ^2 criterion. The empirical value of the criterion was calculated based on the final control results of the students of the experimental and control groups at the beginning of the experiment (Figure 1). The empirical value of the criterion is approximately equal to 0.3. For four levels of knowledge the critical value of χ^2 is approximately equal to 7.8. Therefore, the empirical value of the criterion is smaller than critical. In accordance with the rule of statistical hypothesis testing it means that the null hypothesis is accepted: the experimental and control groups are homogeneous in terms of success in higher mathematics at the beginning of the experiment.

During the second semester students' education was conducted in a distance and blended forms. Students of the control group studied according to the traditional method, and in the process of teaching higher mathematics to the students of the experimental group, an experimental methodological scheme was implemented, which means a change in the structure of the educational material according to the principles of programmed learning, the use of elements of the “flipped classroom” technology in the organization of educational classes, as well as visualization of the educational material at the stage of generalization and systematization of students' knowledge. As mentioned above, students of the experimental and control groups studied according to the same program, the content of the higher mathematics course contained the same topics, and the number of hours for their study was the same. In order to check the effectiveness of the developed methodological recommendations, we compared the final control results in higher mathematics of the students of the experimental and control groups at the end of the second semester (May 2023) after the experimental training. We want to note that the theoretical questions and tasks for the final control results corresponded to the curriculum and were the same for both groups. The distribution of students by levels of mathematical knowledge according to the final control results in higher mathematics of the experimental (EG) and control (CG) groups after the experiment is presented in Figure 2.

Based on the final control results of the students of the experimental and control groups after the experimental training (Figure 2), the empirical value of the criterion χ^2 was calculated, which is approximately equal to 10.1. Therefore, according to the rule of statistical hypothesis testing, an alternative hypothesis is accepted. It can be concluded that after the implementation of the experimental method the results of the final examination in mathematics of the students of the experimental group are significantly better compared to the results of the students of the control group.

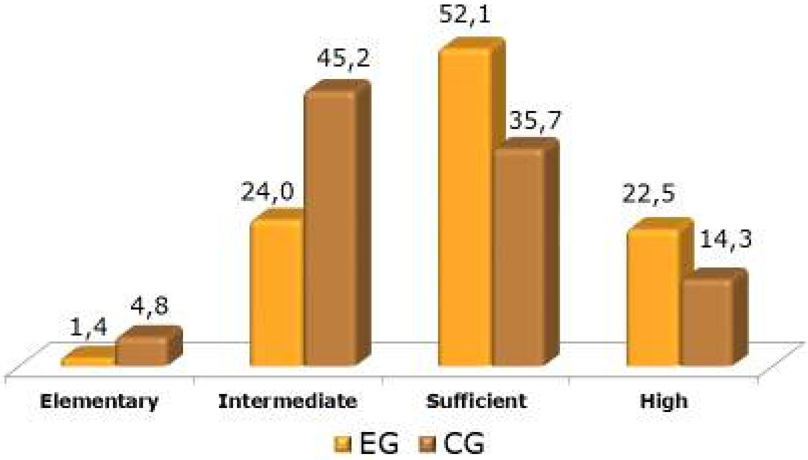

Fig.2. Students' distribution by levels of mathematical knowledge acquisition according to the final control results in higher mathematics of the experimental (EG) and control (CG) groups after the experiment

As mentioned above, the students of both the control and experimental groups studied according to the same program, there were the same number of study hours, the same conditions of study, so we believe that the improvement of academic performance among the students of the experimental group can be explained precisely by the implementation of the methodical recommendations proposed by the authors.

Conclusion. The problem of mathematical training of future specialists of various fields is relevant for many countries of the world. Studying mathematics causes difficulties for many students, which increase in crisis conditions. The search for methodical approaches to high-quality teaching of higher mathematics students continues.

As a result of a theoretical analysis of various educational technologies and our own experience of teaching higher mathematics, we made conclusions about the effectiveness of a methodical approach that combines elements of three technologies, such as programmed learning, the "flipped classroom" and visualization of educational material. The effectiveness of the proposed methodological recommendations was experimentally substantiated using the statistical hypothesis testing method (the χ^2 criterion was used).

We would like to mention that after conducting the research, the students of the experimental group expressed a desire to study other subjects according to the same scheme. It was noted that it is much easier to learn mathematical facts if you structure the educational material, process it in small portions, and compile tables for systematization. They felt more confident, optimistic, satisfied with the results and determined to continue their studies.

The results of our research are the basis for further searches for effective methodological approaches for teaching mathematics to students of non-mathematical specialties. In our opinion questions related to the psychological regularities of the learning process, in particular the development of cognitive interest, positive educational motivation of students, taking into account their emotional state, require further research.

References:

1. About the announcement of the 2020/2021 academic year as the Year of Mathematics Education in Ukraine: Presidential Decree № 31/2020 (2020, January 30). *Uriadovyi kurier*. 2020. №20. [in Ukrainian] URL: https://www.president.gov.ua/documents/312020-32165. Accessed 21 July 2023.

2. Al Mamun, M.A., Azad, M.A.K., Al Mamun, M.A. *et al.* (2022). Review of flipped learning in engineering education: Scientific mapping and research horizon. *Educ Inf Technol*, 27, 1261–1286. https://doi.org/10.1007/s10639-021-10630-z.

3. Bishop, A. J. (1989). Review of research on visualization in mathematics education. *Focus on Learning Problems in Mathematics,* 11(1), 7–16.

4. Buzan, T., Buzan, B. (1993). *The Mind Map Book: How to Use the Radiant Thinking to Maximize Your Brain's Untapped Potential*. Penguin Book Ltd., London.

5. Cioca, L., Nerișanu, *R*. (2020). Enhancing creativity: Using visual mnemonic devices in the teaching process in order to develop creativity in students. *Sustainability, 12*(5), 1985. https://doi.org/10.3390/su12051985.

6. EDUCAUSE: Flipped Classroom. URL: https://library.educause.edu/topics/teaching-and-learning/flipped-classroom. Accessed 21 July 2023.

7. Elkin, D. H. (1965). Pro psykholohichni peredumovy prohramovanoho navchannia [About the psychological prerequisites of programmed learning]. *Radianska shkola*, 12, 22–26. [in Ukrainian]

8. Evagorou, M., Erduran, S., Mäntylä, T. (2015). The role of visual representations in scientific practices: from conceptual understanding and knowledge generation to 'seeing' how science works. *IJ STEM Ed*, 2, 11. https://doi.org/10.1186/s40594-015-0024-x.

9. Fernández-Martín, F.-D., Romero-Rodríguez, J.-M., Gómez-García, G., Navas-Parejo, M.R. (2020). Impact of the Flipped Classroom Method in the Mathematical Area: A Systematic Review. *Mathematics*, 8 (12), 2162. https://doi.org/10.3390/math8122162.

10. Fitsula, M.M. (2009). *Pedahohika* [*Pedagogy*]. Kyiv: Akademvydav. [in Ukrainian]

11. Fornons, V., Palau, R., Santiago, R. (2021). Secondary school students' perception according to their learning style of a mathematics Flipped Classroom. *Journal of Technology and Science Education,* 11 (2). https://doi.org/10.3926/jotse.1092.

12. Fung, C.-H., Besser, M., Poon, K.-K. (2021). Systematic Literature Review of Flipped Classroom in Mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(6), em1974. https://doi.org/10.29333/ejmste/10900.

13. van Garderen, D. (2006). Spatial Visualization, Visual Imagery, and Mathematical Problem Solving of Students With Varying Abilities. *Journal of Learning Disabilities*, 39(6), 496–506. https://doi.org/10.1177/00222194060390060201.

14. Hartikainen, S., Rintala, H., Pylväs, L., Nokelainen, P. (2019). The concept of active learning and the measurement of learning outcomes: A review of research in engineering higher education. *Education Sciences,* 9(4),276.

15. Ianchenko T. V. (2016). Prohramovane navchannia yak rezultat evoliutsii idei pedolohii ta bikhevioryzmu [Programmed learning as a result of the evolution of the ideas of pedology and behaviorism]. *Molodyi vchenyi*, 12(39), 550-553. [in Ukrainian]

16. Ivanova, H.I., Lavrentieva, O.O., Eivas, L.F., Zenkovych, I.O., Uchitel, A.D., (2020). The students' brainwork intensification via the computer visualization of study materials. *CTE Workshop Proceedings,* 7, 185–209. https://doi.org/10.55056/cte.346.

17. Kostiuk H. S. (1964). Pro psykholohichni osnovy prohramuvannia navchannia [About the psychological foundations of teaching programming]. *Radianska shkola*, 5, 54–62. [in Ukrainian]

18. Kotsopoulos, D., Weatherby, C., Woolford, D.G. (2022). Using guided notes to support learning in first-year calculus. *International Journal of Mathematical Education in Science and Technology*, 53(6), 1629-1644.

19. Koval, T. I., Besklinska, O. P. (2020). Use of Visualization Tools for Creation of Electronic Educational Resources in the Process of Learning Mathematical Disciplines in Institutions of Higher Education. *Information technologies and teaching aids,* Volume 77, No. 3. https://doi.org/10.33407/itlt.v77i3.3411.

20. Krutko O. (2010). Chasopys «Radianska shkola» pro prohramovane navchannia v Ukraini v 60-ti roky XX st. [«Soviet School» magazine about programmed education in Ukraine in the 60s of the 20th century]. *Istoryko-pedahohichnyi almanakh*, 1, 15–19 [in Ukrainian]

21. Landa L. N. (1967). Diahnostyka i prohramovane navchannia [Diagnostics and programmed learning]. *Radianska shkola*, 2, 45–51. [in Ukrainian]

22. Lavrenova M.V. (2019). Mentalni karty yak novatsii v osvitnomu protsesi [Mental Maps as Innovations in the Educational Process]. *Scientific Bulletin of Mukachevo State University, Series «Pedagogy and Psychology»*. Issue 1 (9), 36-40. [in Ukrainian]

23. Lo, C.K., Hew, K.F. (2020). A comparison of flipped learning with gamification, traditional learning, and online independent study: the effects on students' mathematics achievement and cognitive engagement. *Interactive Learning Environments,* 28(4), 464-481. https://doi.org/10.1080/10494820.2018.1541910.

24. Lo, C.K., Hew, K.F., Chen, G. (2017). Toward a set of design principles for mathematics flipped classrooms: A synthesis of research in mathematics education. *Educational Research Review*, Volume 22, 50-73. https://doi.org/10.1016/j.edurev.2017.08.002.

25. Macnab, J.S., Phillips, L.M., Norris, S.P. (2012). Visualizations and Visualization in Mathematics Education. In: Norris, S.P. (eds) Reading for Evidence and Interpreting Visualizations in Mathematics and Science Education. Sense Publishers, Rotterdam. https://doi.org/10.1007/978-94-6091-924-4_6.

26. Mashbyts, Yu. I. (1968). Mistse prohramovanoho navchannia u navchalnomu protsesi [The place of programmed learning in the educational process]. *Radianska shkola*, 8, 93–96 [in Ukrainian].

27. *Methods of programmed learning. Basic methodical means of programmed learning.* URL: https://wikipage.com.ua/1x9c99.html. Accessed 21 July 2023.

28. Pereira, L., Gomes, S. (2022). The Impact of Distance Learning on the Teaching-Learning Process of Mathematics in Higher Technical Education. *Journal of Educators Online*, 19(2).

29. Petrillo, J. (2016). On flipping first-semester calculus: A case study. *International Journal of Mathematical Education in Science and Technology* ,Volume 47, Issue 4, 573 – 582. https://doi.org/10.1080/0020739X.2015.1106014.

30. Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. https://www.researchgate.net/publication/241301299_Research_on_visualization_in_learning_and_teaching_mathematics

31. Pressey, S. L. (1927). A machine for automatic teaching of drill material. *School and Society*, 25, 549–552.

32. Pressey, S. L. (1926). A simple apparatus which gives tests and scores and teaches. *School and Society*, 23, 373–376.

33. Rozenberh, M. Y. (1965). Eksperymentalni doslidzhennia efektyvnosti prohramovanoho navchannia [Experimental studies of the effectiveness of programmed learning]. *Radianska shkola*, 8, 25–30. [in Ukrainian]

34. Rozumenko, A.O., Rozumenko, A.M. (2014). *Povtoriuiemo ta systematyzuiemo shkilnyi kurs matematyky: Navchalnyi posibnyk dlia uchniv starshykh klasiv serednikh zahalnoosvitnikh shkil, abituriientiv ta studentiv pershykh kursiv vyshchykh navchalnykh zakladiv* [*We repeat and systematize the school mathematics course: Study guide for students of senior classes of secondary comprehensive schools, entrants and first-year students of higher educational institutions*]. Sumy: SumyDPU named after A. S. Makarenko. [in Ukrainian]

35. Schoenherr, J., Schukajlow, S. (2023). Characterizing external visualization in mathematics education research: a scoping review. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-023-01494-3.

36. Semenikhina, O., Kudrina, O., Koriakin, O., Ponomarenko, L., Korinna, H., Krasilov, A. (2020). The Formation of Skills to Visualize by the Tools of Computer Visualization. *TEM Journal* Volume 9, Issue 4, 1704-1710, ISSN 2217-8309. https://doi.org/10.18421/TEM94-51.

37. Sen, E.O. (2022). Thematic Analysis of Articles on Flipped Learning in Mathematics Education. *Turkish Online Journal of Distance Education-TOJDE,* April 2022, ISSN 1302-6488, Volume 23 Number 2, Article 13, 202-222.

38. Skinner, B. F. (1954). The science of learning and art of teaching / B. F. Skinner // *Harvard Education Review,* 24, 86–97.

39. Skinner, B. F. (1957). *Verbal Behavior*. – New York: Appleton century crofts.

40. Souto, V.T. (2014). Interactive Visualizations in Learning Mathematics: Implications for Information Design and User Experience. In: Marcus, A. (eds) Design, User Experience, and Usability. User Experience Design for Diverse Interaction Platforms and Environments. DUXU. *Lecture Notes in Computer Science*, vol. 8518. Springer, Cham. https://doi.org/10.1007/978-3-319-07626-3_44.

41. Stanberry, M.L., Payne, W.R. (2023). Teaching Undergraduate Calculus at an Urban HBCU through a Global Pandemic. *International Journal of Education in Mathematics, Science and Technology*, 11 (2), 340-357.

42. Vale, I., Barbosa, A. (2023). Visualization: A Pathway to Mathematical Challenging Tasks. In Roza Leikin (Ed.), *Mathematical Challenges For All* (pp.283-306). Springer International Publishing. https://doi.org/10.1007/978-3-031-18868-8.

43. Weurlander, M., Cronhjort, M., Filipsson, L. (2017). Engineering students' experiences of interactive teaching in calculus. *Higher Education Research and Development*, 36(4), 852-865.

44. Zacharopoulos, G., Sella, F., Kadosh, R.C. (2021). The impact of a lack of mathematical education on brain development and future attainment *PNAS.* https://doi.org/10.1073/pnas.2013155118.

45. Zorzos, M., Avgerinos, E. (2023). Research on visualization in probability problem solving. *Eurasia Journal of Mathematics, Science and Technology Education,* 19(4), em2247. https://doi.org/10.29333/ejmste/13066.