Rotor Dynamics and Stability of the Centrifugal Pump CPN 600-35 for Nuclear Power Plants

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Abstract. The paper ensures the vibration reliability of the centrifugal pump CPN 600-35 for the water supply of an industrial circuit at nuclear power plants by improving its technical designs. The main aim of the research is to develop an approach for parameter identification of rotor dynamics and analyze the dynamic stability of the rotor movement. For this purpose, the modified design of the centrifugal pump CPN 600-35 was developed. Also, the main parameters of the rotor dynamics model (e.g., equivalent stiffness and discrete mass) were evaluated based on the parametric identification approach. Moreover, the eigenfrequencies and the corresponding mode shapes of free oscillations were obtained based on the finite element method. Finally, the dynamic stability of the rotor movement was studied based on the developed mathematical model of its oscillations considering the circulating and internal friction forces. Finally, based on the Routh-Hurwitz criterion, the stability region of rotor movement in terms of the dimensionless frequency and friction coefficient was analytically obtained.

Keywords: Energy efficiency · Oscillations · Critical frequency · Discrete-mass model · Parameter identification · Circulating force · Internal Friction · Routh-Hurwitz criterion · Industrial growth

1 Introduction

Ensuring centrifugal machines' energy efficiency and vibration reliability is one of the primary tasks in their design and operation [1]. This problem is to provide rotation stability and a sufficiently low vibration level. It is aggravated by the presence in power machines not only centrifugal forces caused by imbalances, with a frequency equal to the rotational speed but also by high-frequency forces (e.g., blade component) with the frequencies many times higher than the rotor speed [2].

Notably, in the case of stability loss, which can be estimated based on a linear model of rotor oscillations, the total amplitudes may not exceed the permissible. Nevertheless, unacceptable self-oscillations occur, which are typical for nonlinear dynamic systems. Their presence can be determined based on spectral analysis [3]. In this case, an inappropriate state of rotor dynamics can be detected using special equipment, which is not always ensured during the operation of power machines.

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According to the mentioned above, the rotor dynamics and stability of centrifugal pumps for nuclear power plants is an urgent problem, particularly considering the internal friction [4]. Its solving is supplemented by up-to-date computational means in their comprehensive combination with the analytical methods.

Therefore, the main aim is to develop a general approach for ensuring the vibration reliability of centrifugal pumps for nuclear power plants.

The scientific novelty of this study is in considering a single-mass model of rotor dynamics. In this model, equivalent mass and stiffness are determined by the parameter identification approach based on the finite element analysis of rotor dynamics [5]. In this case, stability conditions are determined by an analytical dependence. Moreover, it should analyze the impact of the main design parameters and operating modes on the dynamic stability of the rotor movement.

2 Literature Review

Recent advancements in rotor dynamics and stability are presented in a number of scientific publications worldwide.

Zhou et al. [6] carried out a multiple-frequency synchronization experiment with a dual-rotor actuation vibration system. As a result, the stability criterion between eccentric rotors was developed based on the Poincare–Lyapunov principle. Kumar and Affijulla [7] studied rotor dynamics for power systems. As a result, a rotor stability monitoring system was proposed. Li et al. [8] investigated leakage and dynamic characteristics for different annular gas seals operating in supercritical turbomachinery. As a result, it was shown that the inlet swirl brake significantly reduces the preswirl velocity, lowering the crossover frequency to maximize the operational frequency range. Chao et al. [9] proposed the hierarchical power flow control methodology in smart grids. As a result, rotor stability with demand-side flexibility was enhanced. Chelabi et al. [10] highlighted advancements in analyzing the spatial accelerating flow in a mixed turbine's rotor. Sokolov et al. [11] proposed the methodology for designing automatic rotary motion electrohydraulic drive to ensure technological equipment's vibration reliability.

Also, Li et al. [12] proposed the methodology of dynamic balancing for magnetic bearings supporting rigid rotors based on extended state observers. Filsoof et al. [13] studied critical aeroelastic modes of a tri-rotor wind turbine. As a result, it was shown that the dynamics of the lower rotors change significantly both in eigenfrequencies and damping ratios. Chen et al. [14] investigated dynamics of the vibration system driven by three homodromy eccentric rotors using control synchronization. As a result, the fundamentals for designing vibration machines using control synchronization were provided. Yashchenko et al. [15] studied the impact of bearing housings on centrifugal pump rotor dynamics.

Additionally, Shrestha and Gonzalez-Longatt [16] carried out a parametric sensitivity analysis of rotor angle stability indicators. As a result, the methodology was proposed to estimate critical fault clearing time, eigenvalue points, damping ratio, frequency deviation, voltage deviation, and generator's speed deviation. Li et al. [17] studied the dynamics and stability of a rotor-bearing system with the bolted-disk joint. Krol et al. [18] investigated the vibration stability of spindle nodes to ensure optimal parameters of the technological equipment. Jiang [19] proposed a comprehensive approach based on finite element analysis and multi-objective optimization to study the dynamics of flexible rotor-bearing systems.

Additionally, Li and Tang [20] simulated a numerically high-speed rolling bearing-dual rotor spindle system through discrete modeling dynamics. Zhang et al. [21] studied dynamic characteristics of a novel pocket damper seal with self-regulated injection. Liu et al. [22] proposed a passivity-based control system for quad-tilt rotor unmanned aerial vehicles. Zhao et al. [23] studied vibration characteristics of the helical gear rotor system considering a mixed modification. Also, Osadchiy et al. [24] developed an integrated technology for manufacturing gear systems. Moreover, Volina et al. [25, 26] and Pylypaka et al. [27] studied the movement of blades in centrifugal machines. Finally, Saeed et al. [28] investigated nonlinear dynamics and motion bifurcations of the rotor active magnetic bearings system with a novel control scheme.

Moreover, a number of scientific results were obtained in enhancing the tribological characteristics of functional materials [29]. Particularly, Tarelnyk et al. [30] developed an up-to-date method for surfacing steel shafts. Also, Martsynkovskyy et al. [31] proposed protecting shafts and couplings. Moreover, Svirzhevskyi et al. [32] analyzed methods for evaluating the wear resistance of the contact surfaces for rolling bearings. Finally, Kotliar et al. [33] proposed an approach for ensuring rotor systems' reliability and performance criteria.

3 Research Methodology

3.1 The Design of the Centrifugal Pump

The multistage centrifugal pump CPN 600-35 ensures the water supply for an industrial circuit at nuclear power plants. Its nominal parameters are feed 600 m^3/h, head 35 m, and operating frequency 1500 rpm.

The pump design must correspond to the following national standards concerning rules and regulations in the nuclear power industry according to the requirements of SE "NNEGC "Energoatom": SOU NAEK 158:2020 "Ensuring technical safety. Technical requirements for the design and safe operation of equipment and pipelines of nuclear power plants with the WWER", 159:2020 "Ensuring technical safety. Welding and surfacing of equipment and pipelines of nuclear power plants with the WWER", 160:2020 "Ensuring technical safety. Quality control of the base metal welded joints and surfacing of equipment and pipelines of nuclear power plants with the WWER"; PNAE G-7-002-86 "Rules of strength calculation for equipment and pipelines of nuclear power plants".

According to the requirements presented above, the following design of the centrifugal pump CPN 600-35 for nuclear power plants (Fig. 1) has been developed within the research project "Fulfillment of tasks of the perspective plan of development of a scientific direction "Technical sciences" Sumy State University" ordered by the Ministry of Education and Science of Ukraine (State Reg. No. 0121U112684).

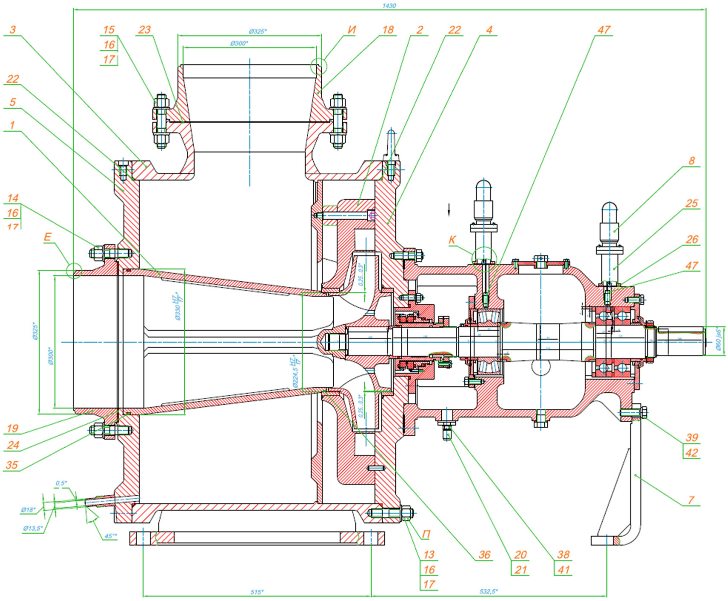


Fig. 1. The assembly of the centrifugal pump CPN 600-35.

The design of the rotor is presented in Fig. 2. It contains a couple of bearing supports. The first one is the spherical double-row rolling bearing SKF 22314 E with the following characteristics: basic dynamic load rating 413 kN; maximum fatigue load 45 kN; maximum rotation speed 4500 rpm. The second one is double back-to-back angular contact ball bearings SKF 7315 BECBJ with the following parameters for each bearing: basic dynamic load rating 104 kN; maximum fatigue load 4.15 kN; maximum rotation speed 5300 rpm.

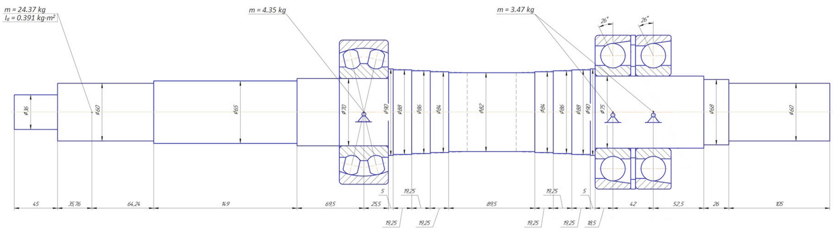


Fig. 2. The assembly (a), the traditional design scheme (b), and the single-mass design scheme (b) of the rotor: m – local masses of bearings, kg; I_d – impeller's diametral moment of inertia, kg·m^2

3.2 Parameter Identification of a Single-Mass Model

The problem of rotor dynamics is mainly studied using the finite element method. However, it does not allow us to evaluate the dynamic stability of rotor systems analytically. Therefore, the parameter identification approach is applied comprehensively with the finite element analysis and analytical approaches.

The procedure of parameter identification of a single-mass mathematical model of rotor dynamics is based on the hypothesis that the compliance of the rotor and its first eigenfrequency can be the same as for the finite-element model. In this regard, the following equations should be satisfied:

$$c_e = \frac{F_{unit}}{x_{res}}; \; m_e = \frac{c_e}{\omega_1^2}, \tag{1}$$

where c_e – equivalent stiffness, N/m; m_e – equivalent mass, kg; F_{unit} – unit force applied at the impeller's mass center at a near-zero value of operating frequency, N; x_{res} – resulting displacement of the impeller's mass center, m; ω_1 – the 1st eigenfrequency.

3.3 The Mathematical Model of the Single-Mass Rotor Dynamics

The proposed model of rotor dynamics in a complex form is as follows (Fig. 3):

$$m_e\ddot{z} + c_e z = D\omega_0^2 e^{i(\omega_0 t+\varphi)} + F_c + F_b + F_q + F_\zeta, \tag{2}$$

where $z = x + i{\cdot}y$ – complex displacement of the mass center; x, y – components of the mass center's displacement in the plane perpendicular to the rotation axis, m; D – permissible residual imbalance, kg·m; ω_0 – operating speed, rad/s; t – time, s; φ – phase shift, rad; i – imaginary unit.

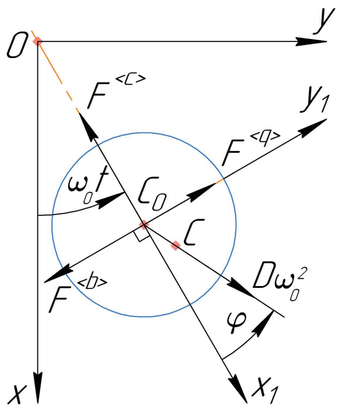


Fig. 3. The design scheme of rotor oscillations: x, y – global coordinates; x_1, y_1 – local coordinates; C_0 – the geometric center of the cross-section; C – the mass center of the rotor.

This model also includes the hydrodynamic forces acting to the impeller, particularly hydrodynamic stiffness force F_c proportional to the displacement, damping force F_b proportional to the velocity, and circulating force F_q proportional to the cross-displacements [34]:

$$F_c = -c_0 z;\; F_b = -b_0 \dot{z};\; F_q = i q_0 z, \tag{3}$$

where c_0 – coefficient of the hydrodynamic stiffness, N/m; b_0 – damping factor, N·s/m; q_0 – coefficient of the circulating force, N/m.

The hydrodynamic coefficients are determined by the following dependencies [35]:

$$c_0 = \frac{\pi d_0 l_0}{4h_0}\Delta p_0;\; b_0 = \frac{\pi \mu d_0 l_0^3}{24 h_0^3};\; q_0 = \frac{1}{2} b_0 \omega_0, \tag{4}$$

where d_0, l_0, and h_0 – diameter, length, and radial gap of the throttle, m; μ – dynamic viscosity of the operating fluid, Pa·s/m; Δp_0 – pressure difference on the gap, Pa.

Additionally, the internal viscous friction force proportional to the relative velocity is considered. Its value in the moving and fixed coordinate systems are as follows [36]:

$$F_{1\zeta} = -\zeta \dot{z}_1;\; F_\zeta = -\zeta(\dot{z} - i\omega_0 z), \tag{5}$$

where ζ – internal friction coefficient, N·s/m.

Overall, the initial differential equation of rotor dynamics (2) takes the form:

$$m_e \ddot{z} + (b_0 + \zeta)\dot{z} + \left[c_e + c_0 - i(q_0 + \zeta\omega_0)\right] z = D\omega_0^2 e^{i(\omega_0 t + \varphi)}, \tag{6}$$

or in projections on the coordinates x and y, and considering formula (4):

$$\begin{cases} m_e \ddot{x} + (b_0 + \zeta)\dot{x} + (c_e + c_0)x + \frac{1}{2}(b_0 + 2\zeta)\omega_0 y = D\omega_0^2 cos(\omega_0 t + \varphi); \\ m_e \ddot{y} + (b_0 + \zeta)\dot{y} + (c_e + c_0)y - \frac{1}{2}(b_0 + 2\zeta)\omega_0 x = D\omega_0^2 sin(\omega_0 t + \varphi). \end{cases} \tag{7}$$

3.4 Dynamic Stability of the Rotor Movement

It is known that internal friction does not affect the amplitude-frequency response. However, the impact of this force on rotor stability was not considered entirely.

The dynamic stability of the rotor's motion can be studied according to the Routh-Hurwitz criterion [37]. In this case, components in the right parts of Eqs. (3) are zero, and the differential operator p is introduced. Consequently, the system of linear algebraic equations can be obtained:

$$\begin{cases} \left[m_e p^2 + (b_0 + \zeta)p + c_e + c_0\right]x + \frac{1}{2}(b_0 + 2\zeta)\omega_0 y = 0; \\ -\frac{1}{2}(b_0 + 2\zeta)\omega_0 x + \left[m_e p^2 + (b_0 + \zeta)p + c_e + c_0\right]y = 0. \end{cases} \tag{8}$$

Therefore, the internal friction increases the terms concerning damping and the circulating forces.

The corresponding characteristic equation is as follows:

$$\left|\begin{bmatrix} m_e p^2 + b_0 p + c_e + c_0 & \frac{1}{2}(b_0 + 2\zeta)\omega_0 \\ -\frac{1}{2}(b_0 + 2\zeta)\omega_0 & m_e p^2 + (b_0 + \zeta)p + c_e + c_0 \end{bmatrix}\right| = \sum\nolimits_{j=0}^{4} a_j p^{4-j} = 0, \tag{9}$$

where the following coefficients have been introduced:

$$\begin{aligned} a_0 &= m_e^2;\ a_1 = 2(b_0+\zeta)m_e;\ a_2 = (b_0+\zeta)^2 + 2(c_e+c_0)m_e; \\ a_3 &= 2(b_0+\zeta)(c_e+c_0);\ a_4 = c^2 + (b_0+\zeta)\zeta\omega_0^2 + \tfrac{1}{4}b_0^2\omega_0^2. \end{aligned} \tag{10}$$

The negative parts of solutions p_j determine the stability region of rotor movement. According to the Routh-Hurwitz criterion, this requirement is completely satisfied for the following inequalities:

$$\begin{cases} a_j > 0; \\ \Delta_2 = a_1 a_2 - a_0 a_3 = 2m_e(b_0+\zeta)\left[(c_e+c_0)m_e + (b_0+\zeta)^2\right] > 0; \\ \Delta_2 a_3 - a_1^2 a_4 = m_e(b_0+\zeta)\left[(c_e+c_0)(b_0+\zeta)^2 - m_e(b_0+2\zeta)^2\omega_0^2\right] > 0. \end{cases} \tag{11}$$

The first two of these inequalities are automatically satisfied. However, the last one leads to the following stability condition (Fig. 4):

$$\psi(\chi) < \frac{2(1+\chi)}{1+2\chi}, \tag{12}$$

where the following dimensionless frequency ψ and friction ratio χ have been introduced:

$$\psi = \frac{\omega_0}{\omega_{cr}};\ \chi = \frac{\zeta}{b_0}, \tag{13}$$

where ω_{cr} – critical frequency,

$$\omega_{cr} = \sqrt{\frac{c_e + c_0}{m_e}} = \omega_1\sqrt{1 + \frac{c_0}{c_e}}. \tag{14}$$

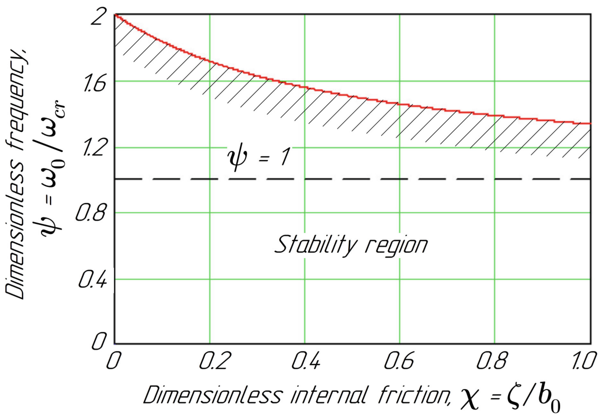


Fig. 4. The stability region of rotor movement.

Therefore, despite the internal friction not affecting the amplitude-frequency response, it impacts the rotor stability. Moreover, according to the stability criterion, the maximum operating frequency should be less than the maximum value $\psi\omega_{cr}$, where the dimensionless coefficient varies in a range of $1 \leq \psi < 2$ depending on the value of the internal friction coefficient ζ. Remarkably, if the dimensionless frequency $\psi < 1$, the rotor motion is stable for a whole range of change in the internal friction coefficient.

4 Results

The initial data for dynamic analysis and stability of the designed centrifugal pump CPN 600-35 based on the finite-element model is summarized in Table 1. The calculations are realized using the authors' operating file "Critical frequencies of the rotor" of the computer algebra system MathCAD.

Table 1. The initial data of the dynamic analysis.

Section no.	Length, m	Diameter, m	Local mass, kg	Moment of inertia, kg·m^2	Stiffness, N/m
1	0.045	0.036	–	–	–
2	0.036	0.060	–	–	–
3	0.064	0.060	24.37	0.391	–
4	0.149	0.065	–	–	–
5	0.07	0.070	–	–	–
6	0.026	0.070	4.35	–	$1 \cdot 10^{12}$
7	0.005	0.090	–	–	–
8	0.057	0.086	–	–	–
9	0.090	0.082	–	–	–
10	0.057	0.086	–	–	–
11	0.005	0.090	–	–	–
12	0.019	0.075	–	–	–
13	0.042	0.075	3.47	–	$1 \cdot 10^{12}$
14	0.053	0.075	3.47	–	$1 \cdot 10^{12}$
15	0.026	0.068	–	–	–
16	0.105	0.060	–	–	–

The traditional design scheme corresponding to Fig. 2 and Table 1 is presented in Fig. 5.

The first three resulting mode shapes of free oscillations are presented in Fig. 6.

The corresponding critical frequencies were obtained using the finite element method using the authors' operating file "Critical frequencies of the rotor" of the computer

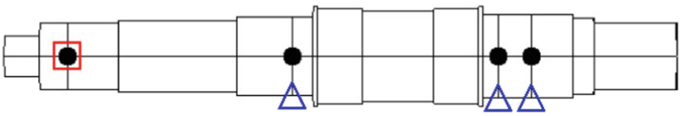


Fig. 5. The traditional design scheme of the rotor: △– bearing support; ●– local mass; □– moment of inertia.

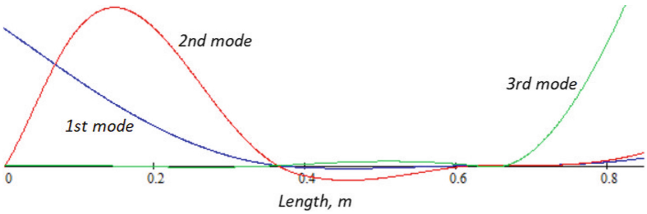
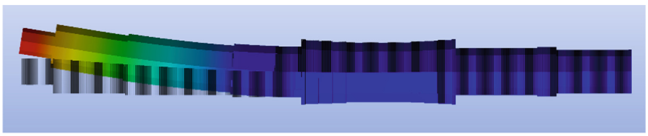


Fig. 6. The mode shapes of free oscillations, obtained using the program "Critical frequencies of the rotor".

algebra system MathCAD. The corresponding values are: $\omega_{cr1} = 702$ rad/s, $\omega_{cr2} = 2525$ rad/s, and $\omega_{cr3} = 9995$ rad/s.

The reliability of this model is proved by the similar results obtained using the ANSYS software (Fig. 7).



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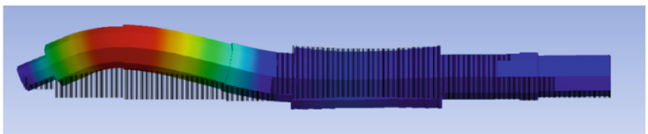

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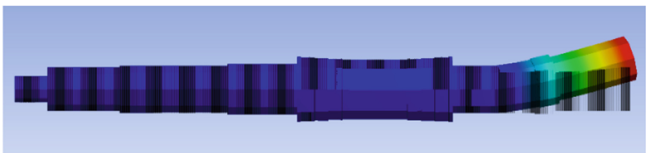

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Fig. 7. The 1st (a), 2nd (b), and 3rd (c) mode shapes of free oscillations, obtained using the ANSYS software.

The comparison of the results obtained using the MathCAD and ANSYS software is presented in Table 2.

Table 2. Comparison of the results.

Software	Eigenfrequency, rad/s		
	1st	2nd	3rd
MathCAD	702	2525	9995
ANSYS	687	2432	8723
Relative error, %	2.2	3.8	14.6

The relative error for evaluating the 1st eigenfrequency ω_1 does not exceed 2.2%. Therefore, the authors' methodology, realized within the operating file "Critical frequencies of the rotor", is reliable for the designed centrifugal pump CPN 600-35 for nuclear power plants.

Notably, since the 1st critical frequency $\omega_{cr1} = 732$ rad/s is 4.47 times higher than the operating speed $\omega_0 = 157$ rad/s, detuning from the resonance is equal to 347%. Therefore, mechanical vibrations can be considered according to the international standard ISO 1940-1:2003 "Mechanical vibration – Balance quality requirements for rotors in a constant (rigid) state – Part 1: Specification and verification of balance tolerances (IDT)".

According to formula (1), the equivalent stiffness can be determined using the finite element method using the authors' operating file "Forced oscillations of the rotor" of the computer algebra system MathCAD. After calculations, the corresponding value is equal to $c_e = 2.02{\cdot}10^7$ N/m. Also, the equivalent mass (1) is equal to $m_e = 41.0$ kg. Also, for the particular case study, the following physical and geometrical parameters of the throttling gaps have been considered: pressure difference $\Delta p_0 = 3.43{\cdot}10^5$ Pa; dynamic viscosity of the operating fluid at normal conditions $\mu = 1.0{\cdot}10^{-3}$ Pa·s. According to the pump design (Fig. 1), the following geometrical parameters of the radial throttle have been considered: diameter $d_0 = 0.2245$ m; length $l_0 = 0.045$ m; gap $h_0 = (0.25\ldots0.30){\cdot}10^{-3}$ m.

According to formulas (4) and (13), the following parameters have been evaluated: damping factor $b_0 = 99.1\ldots171.4$ N·s/m.

Due to the results presented by Roy and Tiwari [38], for a similar rotor, and under the common assumption that the variation coefficient is equal to 0.2, the friction coefficient in a three-sigma range is equal to $\zeta = 8\ldots32$ N·s/m. According to formula (14), the dimensionless friction ratio $\chi = 0.047\ldots0.323$, and the dimensionless frequency $\psi = 0.180\ldots0.186$.

Finally, the maximum dimensionless frequency (12) is equal to $\psi_{max} = 1.608$. Therefore, since this value is 8.64 times higher than the value $\psi = 0.186$, the rotor motion is stable with a margin of 764%.

5 Conclusions

Thus, in the paper, the centrifugal pump CPN 600-35 for the water supply of an industrial circuit at nuclear power plants has been modernized. This design corresponds to national standards concerning rules and regulations in the nuclear power industry according to SE "NNEGC "Energoatom" requirements, i.e., SOU NAEK 158:2020, 59:2020, 160:2020, and PNAE G-7-002-86.

For ensuring vibration reliability of the designed pump, the parameter identification approach has been applied jointly with the finite element analysis and analytical modeling. As a result, the equivalent mass and stiffness of the rotor system have been evaluated. The mathematical model of rotor dynamics has been developed considering inertia, stiffness, damping, circulating, and internal friction forces.

Since the operating speed of 157 rad/s is significantly less than the first critical frequency of 702 rad/s, detuning from the resonance equals 347%. Therefore, dynamic balancing of the rotor should be carried out according to the international standard ISO 1940-1:2003.

Moreover, the first three eigenfrequencies have been calculated numerically using the ANSYS software and the developed operating file "Critical frequency of the rotor" of the computer algebra system MathCAD. Each program is based on the finite element method. The relative difference between the first two eigenfrequencies does not exceed 4%.

Finally, based on the Routh-Hurwitz criteria, the dynamic stability of the rotor movement has been assessed in terms of operating frequency and friction coefficient, and the corresponding stability region has been built. Notably, for the designed rotor, its motion is dynamically stable with a margin of 764%.

Notably, the presented research corresponds to the objective "Increasing the pressure of the stages of pumping units and ensuring the vibration reliability of the functional elements of the complex hydrodynamic system based on improving the design of pumping equipment of nuclear power plants (NPPs) by developing technical designs for water supply pumps and auxiliary systems" according to the Agreement No. BF/26-2021 between Sumy State University and Ministry of Education and Science of Ukraine.

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