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Research paper

The Issue of Determination of the Rigidity Characteristics of Reinforced Concrete Elements with Normal Cracks

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Abstract

Most of the reinforced concrete slab structures are statically indeterminate systems. In these systems, the redistribution of internal forces depends on the nature redistribution of rigidities between their separate elements. The presence of cracks significantly affects the change of elements rigidity of reinforced concrete structures. In the plate-ribbed systems, which include bridge structures, ribbed prefabricated and monolithic slabs, at the moment when normal cracks are wide enough, spatial torsion cracks may be absent. In this article the method of analysis of the rigidity of reinforced concrete elements with normal cracks is presented. The method is based on approximate striping of cross section to separate lines. The method is approximate, but it is rather convenient, since it can be used as a subprogram for analysis of complex statically indeterminate reinforced concrete systems. The torsional rigidity of every rod of the system is determined in such a subprogram at iterative analysis in automatic mode. As a result of this analysis, the forces in the rods of statically indeterminate system s are determined more accurately as the change of flexural as well as torsional rigidities caused by normal cracks formation were taken into account. The comparison of the results obtained by the torsional rigidity determination method with the results obtained by the numerical method confirmed the developed methodology

Keywords: *normal crack; reinforced concrete ribbed slab; torsion; torsional rigidity*

1. Introduction

The category of plate-ribbed systems includes bridge structures, monolithic and prefabricated ribbed overlaps. The normal cracks arise at the edges of these structures cause of bending moments. It often happens that when a sufficiently wide disclosure normal of cracks, the spatial fracture from torsion are not available. At the same time, the load redistribution between contiguous ribs and between separate precast elements overlap depends not only on the bending, but also by torsional rigidity ribs [1].
Experimental research [2] shows that rigidity of ribs prefabricated slabs on the torsion changes during crack formation. It should be noted that much theoretical and experimental work was devoted to the study of bending rigidity despite the great importance of bending as well as torsional rigidity in forces redistribution in statically indeterminate systems. These works include works [3, 4, 5, 6], and others. Torsional rigity study is limited to a small number of scientific works.
Existing methods of determination of torsional rigidity [7, 8] concern only the reinforced concrete elements with spatial (spiral) cracks. The torque is applied to the part of the end surface of the rectangular element [9].The main objective in determining the torsional rigidity is calculating the displacements in the end of the rectangular element. If you use the "two-rod" model [9] may suffer the accuracy of calculations. Therefore, the aim of this article is improvement the methods of determination the displacements of the reinforced concrete element of rectangular section with normal cracks. The end element is loaded with torque on the part of the section.

2. Main part

The analytical model is a reinforced concrete girder element of rectangular cross section. Element is divided into blocks with normal cracks, which arise from the action of bending moment. The torque is applied to the top part of the end surface of the girder (zone compressed by bending).
The results are used for numerical and analytical method of research. We used differential equations and methods of differential and integral calculus.
Consider a concrete element with normal cracks (fig. 1).

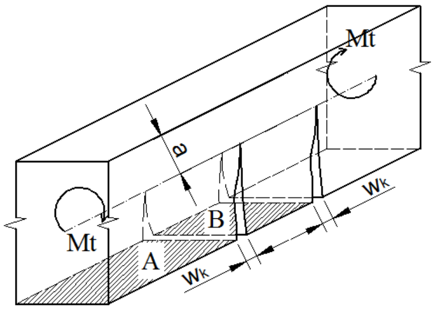

Fig. 1: Scheme of reinforced concrete element with normal crack, which is loaded torque

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Transfer of torque from *A* block to block *B* (fig. 1) occurs through the compressed zone of concrete. To determine the rigidity of the reinforced concrete element with normal cracks under the action of torque is required to determine the displacement the block *A* relative to block *B*.(fig. 2).

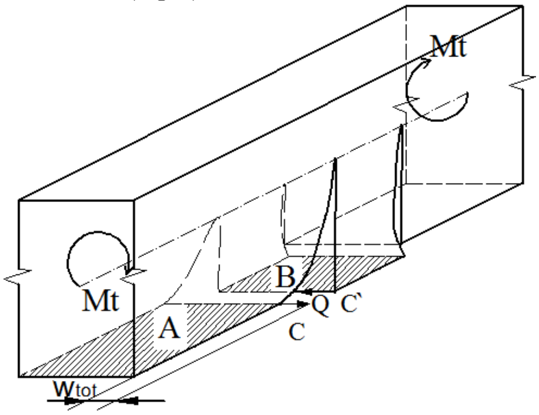

Fig. 2: Displacement of block *A* relatively to block *B*, separated by a normal crack

Application scheme of torque to the block *B* looks as shown in (fig. 3).
Determination of the torsional rigidity of a reinforced concrete element with a normal (caused by bending) crack can be represented in the following sequence:
1. To create a static definability, one should visualize the dissection of the longitudinal reinforcement in the fracture.
2. Determine the displacement of block A relatively to block B, separated by a normal crack of one block relative to the other. In the compatibility conditions of deformations in the dissected reinforcement determine the nudge force *Q* in it (Figure 2). Taking into account the nagel force *Q* and the external torque *Mt*, we determine the real horizontal displacement in the fracture $a_{to}t$ of one block relatively to the other (Figure 2).
3. Determine the angle of rotation of the fictitious conditionally continuous element φ_{ekv} as the ratio of the previously determined displacement a_{tot} (point *4*) to the turning radius, (approximately half the height of the rod section).
4. Determine the torsional rigidity of the element with crack *Bt* according to the formula:

$$B_t = \frac{M_t \cdot l}{\varphi_{ekv}}$$

To determine the relative motion of blocks, it is necessary to determine the stresses in the longitudinal sections of the element, to a part of the cross section to which a torque is applied. **Fig. 2**:
The task of the elasticity theory about torsion rod of rectangular cross section offers a solution based on these assumptions:
- end of the rod is uniformly loaded by the tangential forces;
- the resultant of these forces is the torque M_t;
- according to the method of application torque on Fig. 3 (on a part of the section) stresses and displacements can not be defined by the formulas torsion.

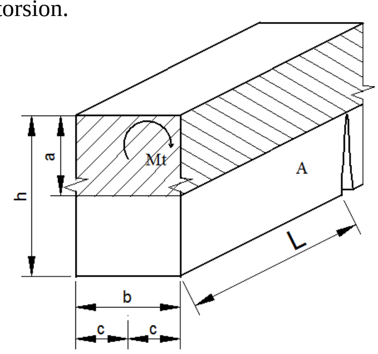

Fig. 3: Scheme of torque transmission through the compressed zone of concrete

This task can be solved using the finite element method (FEM) using volumetric finite element (FE). There are difficulties in using these elements. Keep in mind that this problem is only part of the solution of the more general task of determining the torsional rigidity of reinforced concrete elements with cracks.
To solve the problem we use the method [9]. The difference will be in the separation the rod into arbitrary number, instead of two.
Let us first consider a rod, cut by a horizontal plane into two linear finite elements – two beams *I* and *II* (fig.4). The length of the rod *L* is the length of the block bounded by cracks.
The cutting plane lies the boundary between the elements *I* and *II*. In this case, the depth of a cross section of the upper rod is equal to the compressed (caused by bending) zone, and the depth of the element *II* is the height of the crack. In Fig. 4 consoles are shown conditionally. In fact, the ends of elements *I* and *II* are in the same vertical plane XOZ.

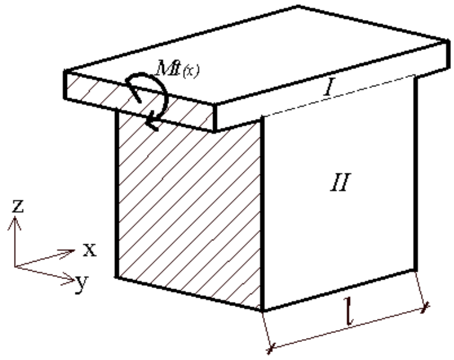

Fig. 4: Dissection scheme of a rod block into two rods. The block is bounded by cracks by both sides

The forces *S(x)* act in the plane of dissection in the vertical plane (fig. 5) and tangents *τ (x)* – in the horizontal plane. The tangential forces are directed along the *y*-axis in Fig. 5. It should be noted that tangential forces directed along the *x*-axis will act in the plane of dissection, and in the approximate method they will be neglected because of their smallness.

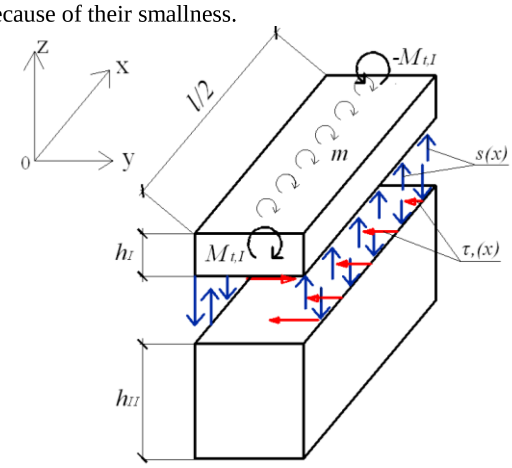

Fig. 5: Stresses acting in the plane of dissection of double-layer console rod.

The block, separated by cracks, is represented in the form of a physical model (fig.5). It can be seen that the fibers of such a "model" subjected by torque, as shown in Fig. 5 are subjected to compression-expansion deformations in the vertical and horizontal directions. Volumetric diagrams of linear vertical (transverse) forces and shear forces acting in the horizontal plane of the rod dissection in Fig. 5 are shown in Fig. 6.

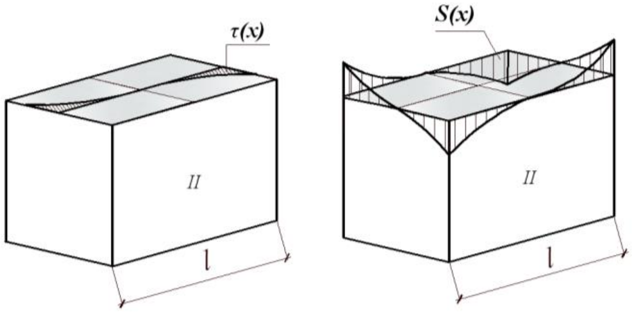

Fig. 6: Diagrams of tangential shear and transverse forces acting in the plane of the dissection of the rod

It is also true that that the dissection of a l – long block by parallel planes in *XOY* plane into the *n-th* number of layers (rod finite elements) gives greater accuracy of the calculation results. The more number of elements of dissection are calculated, the higher is the accuracy of determination of unknown forces and deformations.

Loading scheme of the block *B* (fig. 1) and its division into separate lanes is as shown in Fig. 7.

We spend ***n*** horizontal sections which are parallel to the plane OXY (fig. 7) and get ***n + 1*** lanes (rods).

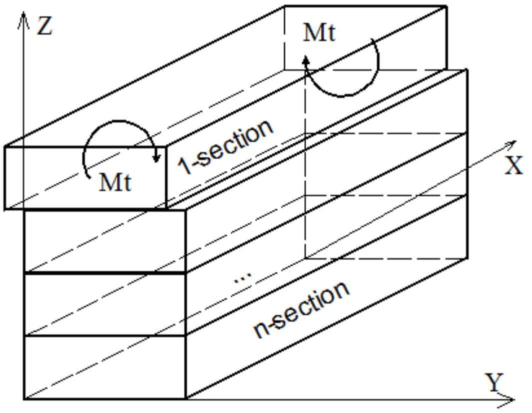

Fig. 7: The scheme of dividing the block into individual strips (rods)

Considering the symmetrical loading block of Fig. 7, the scheme of loading the *i- rod*, bearing in mind the analogy with [9] can be represented as shown in Fig. 8.

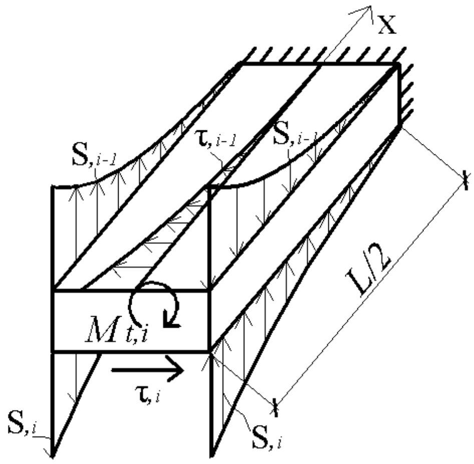

Fig. 8: Scheme of the internal forces applied to the i-rod

Unknown $S_i(x)$ and $\tau_i(x)$ is determined by the joint of deformations community in the *i*-section (similar to [9]). Typical strings of the system of equations to determine the unknown forces will look like:

$$\left.\begin{aligned}
&-M_{t,i}\cdot\frac{r_i}{GJ_i}+T_{i-1}\cdot\frac{r_i^2}{GJ_i}+T_i\cdot\frac{r_i^2}{GJ_i}-QS_{i-1}\cdot\frac{b\cdot r_i}{GJ_i}+\\
&+QS_i\cdot\frac{b\cdot r_i}{GJ_i}+T_i''\cdot\frac{r_i^2}{G\cdot b}=M_{t,i+1}\cdot\frac{r_{i+1}}{GJ_{i+1}}-T_i\cdot\frac{r_{i+1}^2}{GJ_{i+1}}-\\
&-T_{i+1}\cdot\frac{r_{i+1}^2}{GJ_{i+1}}+QS_i\cdot\frac{b\cdot r_{i+1}}{GJ_{i+1}}-QS_{i+1}\cdot\frac{b\cdot r_{i+1}}{GJ_{i+1}}-\\
&-T_i''\cdot\frac{r_{i+1}}{G\cdot b};\\
&M_{t,i}\cdot\frac{C}{GJ_i}-T_{i-1}\cdot\frac{r_i\cdot C}{GJ_i}-T_i\cdot\frac{r_i\cdot C}{GJ_i}+QS_{i-1}\cdot\frac{b\cdot C}{GJ_i}-\\
&-QS_i\cdot\frac{b\cdot C}{GJ_i}-QS_i''\cdot\frac{r_i}{EF}=M_{t,i+1}\cdot\frac{C}{GJ_{i+1}}-T_i\cdot\frac{r_{i+1}\cdot C}{GJ_{i+1}}-\\
&-T_{i+1}\cdot\frac{r_{i+1}\cdot C}{GJ_{i+1}}+QS_i\cdot\frac{b\cdot C}{GJ_{i+1}}-QS_{i+1}\cdot\frac{b\cdot C}{GJ_{i+1}}+QS_i''\frac{r_{i+1}}{EF};
\end{aligned}\right\}\qquad(1)$$

System (1) shall be compiled for each «*k*» the seam (longitudinal section).

Consequently, the number of equations is equal to *2n*, where *n* is the number of sections (fig. 7).

In the expression (1) is indicated:

$T_i=T_i(x)$ – the summary tangential forces which relate with the linear tangential forces $\tau_i(x)$ by the differential ratio:

$$T_i^I(x)=\tau_i(x) \qquad (2)$$

$QS_i=QS_i(x)$ – the summary vertical forces which relate with the linear tangential forces $Si(x)$ by the differential ratio:

$$QS_i^I(x)=S_i(x) \qquad (3)$$

r_i – half the thickness of the *i-th* rod;

b – width of the cross section of the rod (see fig. 3.);

$C=b/2$ – rod turning radius (half the width of the section);

GJ_i – torsional rigidity of *i-th* rod

EF – rigidity of conditional rods of unit width, which simulate compression (stretching) the fiber of rods in the vertical direction [9].

The system of equations (1) can be conveniently solved by decomposing the unknown members in the Fourier series by cosines:

$$\begin{aligned}
&T=\sum_{n=1}^{\infty}T_n\cdot Cos(\alpha\cdot x)',\\
&QS=\sum_{n=1}^{\infty}Q_n\cdot Cos(\alpha\cdot x)'
\end{aligned}\qquad(4)$$

where $\alpha=\pi\cdot n/l$.

To solve a system of equations, external torque M_t also is decomposite in series by cosines:

$$M_t(x)=\sum_{n=1}^{\infty}M_{t_n}\cdot Cos(\alpha\cdot x)' \qquad (5)$$

where $M_{t,n}$ – the Fourier coefficient, which indicates the decomposition of the external moment in the Fourier series. This Fourier coefficient is determined simply enough. The character of change of function of the external torque along the length of rod does not affect its definition.

Then it is necessary differentiate, expanded, reduced to the $Cos(\alpha{\cdot}x)$ all unknown and load terms in Fourier series. So, instead of differential equations get a system of linear algebraic equations. In the case of the same cross-section of all t rods (when the rod is divided into separate layers of equal thickness) will be:

$$\left.\begin{aligned}&T_{i-1}\cdot\frac{r_2}{GJ}+T_i\left(\frac{2r^2}{GJ}+\frac{2\alpha^2 r}{G\cdot b}\right)+T_{i+1}\frac{r^2}{GJ}+QS_{i-1}\left(-\frac{b\cdot r}{GJ}\right)+\\&+0+QS_{i+1}\frac{b\cdot r}{GJ}=\frac{r}{GJ}(M_{t,i}+M_{t,i+1});\\&T_{i-1}\left(-\frac{r\cdot C}{GJ}\right)+0+\mathrm{T}_{i+1}\frac{r\cdot C}{GJ}+QS_{i-1}\frac{b\cdot C}{GJ}+\\&+QS_i\left(-\frac{2bC}{GJ}-\frac{2\alpha^2 r}{EF}\right)+QS_{i+1}\frac{b\cdot C}{GJ}=\frac{C}{GJ}(M_{t,i+1}-M_{t,i});\end{aligned}\right\}\quad(6)$$

where $r_{i+1}=r_i=r;\ GJ_i=GJ_{i+1}=GJ$.

In the expression (6) through T_i and QS_i are designated unknown expansion coefficients in the Fourier series of cosine (4), and through $M_{t,i}$ – the Fourier coefficients in the expansion of external torque M_t moments in the series (5).

The system of equations (6) is solved ***m*** times, where ***m*** is the upper limit of the summation of series (4) and (5). Usually 7–9 odd term numbers are enough to provide acceptable calculation accuracy.

After determining forces $Ti(x)$ and $QSi(x)$ a rod is considered as loaded by external torque $Mt,i(x)$ and the forces $T_{i-1}(x)$, $Ti(x)$; $QS_{i-1}(x)$, $QSi(x)$, defined by solving the equations system (fig. 8).

If known the forces $Ti(x)$ and $QSi(x)$, it is easy to determine the upper part of the block movements relative to its lower part (fig. 1).

After determination of the unknown forces $T(x)$ and $S(x)$, we determine the angle of rotation of the upper part of the block relatively to its lower part. The rotation of the block to which the torque Mt is applied relatively to the adjacent block is resisted not only by the compressed zone (non-cracked section), but also by the reinforcement. Now we examine displacement Δ to X axis of considering block relatively to adjacent one**.**

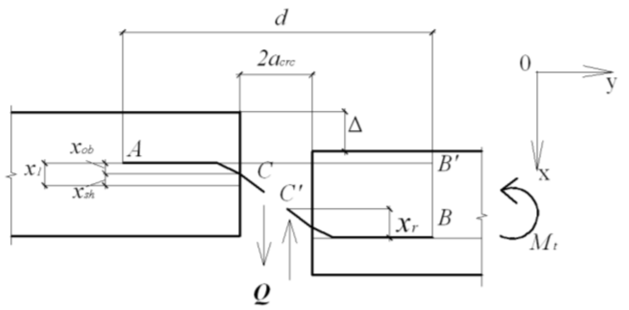

Fig.9: Scheme of deformation of reinforcement and mutual rotation of blocks

In Fig. 9 the following designations have been adopted:

- *2acrc* – the width of the crack;
- X_{sh} – the displacement of the cutting point of the reinforcing bar by the shift of the latter (in general case caused by shear and bending, but considering the small value of a_{crc}, the shear deformations mainly predominate);
- X_{ob} – the displacement caused by the caving of concrete while considering the work of reinforcement in concrete as a rod on an elastic foundation, the role of which is performed by a concrete shell.

After that the unknown transverse (nagel) force Q is determined in the armature of element like [9] and [2].

It is determined from the condition that the horizontal C and C^I point's displacements (fig. 9) are equal in a place of mental cutting an armature.

$$Q=\frac{a_{Mt}^{ver}-a_{Mt}^{nig}}{2\cdot a_{ob,ed}+2\cdot a_{sh,ed}+a_{Q,ed}^{ver}-a_{Q,ed}^{nig}},\quad(7)$$

where we have marked:

$a_{ob,ed}$; $a_{sh,ed}$ – the displacements from the concrete crimping and armature shear from the unit force $\overline{Q}=1$ action; these displacements are defined as the displacements of the rod, which is based on a continuous elastic base [9];

a_{Mt}^{ver} – the point C displacement from the torsion of the upper part, i.e. of the compressed zone (fig. 9) by external torque M_t considering the internal forces $QS_i(x)$ and $T_i(x)$;

$a_{Q,ed}^{ver}$ – the point C displacement from the torsion of the upper part, (fig.9) by external torque, generated by a unit force in the armature $\overline{Q}=1$;

a_{Mt}^{nig} – the point C' displacement, i.e. the lower part in Fig. 9, from the action of internal forces $QS_i(x)$ and $T_i(x)$, that arise as a result of torsion by a external moment M_t;

$a_{Q,ed}^{nig}$ – the point C' displacement, i.e. the lower part in Fig. 7, from the action of internal forces $QS_i(x)$ and $T_i(x)$, that arise as a result of torsion by a unit force $\overline{Q}=1$.

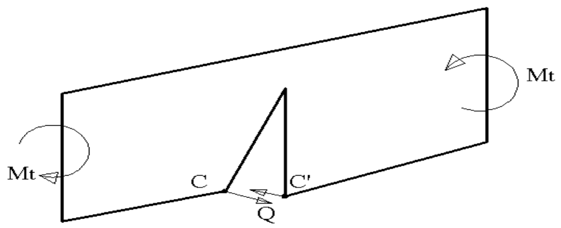

Fig. 9: The scheme of the mutual rotation of two blocks, separated by crack

The components of the displacement in the expression (7), are defined as in [2, 9] but with the changes, which connected with the definition of internal forces, made in this article.

After calculating the unknown quantity Q you can determine the real displacement in the crack w_{tot}.

Order to determine torsional rigidity of the element with normal crack should identify the rotation angle of conventionally continuous (without cracks) element:

$$\varphi_{ekv}=\frac{w_{tot}}{h/2}.\quad(8)$$

The ratio of the rotation angle of continuous element without cracks to an equivalent, which defined by (8), gives us the ratio of the continuous element rigidity to the rigidity of the element with normal crack.

Use of a multilayer scheme (fig. 3) advantageously differs from a two-layer scheme [9], because accuracy of determining the forces grows just as in FEM with decreasing the finite element size is increased the accuracy of the result.

Thus, for a beam with cross section b x h = 10 x 20 cm, with block length between cracks L = 20 cm and with depth of the compressed zone 4 cm, the maximum forces value $S(x)$ in the end of the element (in the cross section with a crack) for the multilayer scheme (when the number of layers is equal to five) is constituted to 36,8 N/cm, and for the two-layer scheme – 28,3 N/cm.

As you can see, the difference in values is a significant. Layers thickness (conditionally – dimensions of finite elements), which are needed to obtain an acceptable accuracy, can be determined by trial calculations.

3. Conclusions

A method for determining the internal forces in rod element was developed. Torque is applied to a portion of the cross section. The

calculation of these forces allows to determine the displacements in the cross section with normal to crack under torque action.
This, in turn, allows to determine the torsional rigidity of reinforced concrete element with normal cracks.
The perspective is the development of methodology for determining the rigidity characteristics of concrete elements of an arbitrary cross-section with normal cracks.
In addition, it should develop a program on the computer for automatic calculation of displacements (and rigidity parameters) of elements with normal cracks. In the future, the program will be used as a subprogram in the calculation of bridges, slabs and other ribbed systems taking into account the spatial work.
More accurately torsional rigidity of reinforced concrete elements with cracks can be defined, using the technique [10]. According to this method, it is proposed to calculate the mutual displacement of the normal crack faces basing on the data processing of plenty of numerical calculations using volumetric finite elements.
However, this procedure is rather cumbersome and at present can not be used as a subprogram for analysis of complex statically indeterminate rod systems.

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