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Improvement of Crop Production Planning in Ukraine Using the Cobb-Douglas Production Function

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► **Abstract.** The relevance of the study is conditioned by the need to improve the process of planning the crop production by agricultural enterprises of Ukraine through the construction of a functional relationship between the results of production activities of enterprises and the factors that determine these results. The purpose of the study is to improve the process of planning crop production by agricultural enterprises of Ukraine through the development of an econometric model in the form of a Cobb-Douglas production function. To model the processes of crop production, it is proposed to use the Cobb-Douglas production function, which describes the dependence of the crop production index on the indices of agricultural land acreage and labour productivity. Based on statistical information for 1996-2020, unknown parameters of the Cobb-Douglas production function were found by regression analysis. The high quality of the constructed model is substantiated. High predictive properties of the model are proved. To analyse the efficiency of resource use in crop production by agricultural enterprises, the coefficients of average, marginal products of labour and capital are calculated. The sensitivity of crop production to changes in crop areas and labour productivity is investigated. It is determined that if the index of agricultural areas is increased by 1%, the index of crop production will increase by 0.77%; and if the index of labour productivity is increased by 1%, the index of crop production will increase by 0.81%. An increasing return on the scale of resources spent is revealed, which indicates an increase in production volumes with an increase in the volume of resources in the crop production industry

► **Keywords:** econometric model, model quality assessment, forecast, resource efficiency, output elasticity, economies of scale, agricultural enterprises

► Introduction

The development of methods for forecasting the production activities of business entities, evaluating and analysing the efficiency of their management using objective dependencies between the results of activities and the factors that determine these results are modern ways of improving the production planning process. Methods for evaluating and analysing the effectiveness of the functioning of business entities can be divided into two groups.

Methods of the first group involve the use of financial and economic coefficients that characterise economic activity. In [1], the advantages and disadvantages of using these methods are indicated. The advantage of using financial and economic coefficients for evaluating and analysing the performance of business entities is the simplicity of their calculation. The disadvantages of using these methods include the following: 1) analysis of an individual coefficient can only evaluate a part of the activity. Assessment of the efficiency of all production activities involves the analysis of a large number of coefficients, which leads to its complication; 2) there is a paradox [2], in which a business entity that has better financial and economic indicators than another has a lower overall efficiency; 3) when comparing the efficiency of business entities whose results differ significantly, relative coefficients are used. They are calculated as the ratio of performance results to resources spent. Thus, the existence of only a stable return on the scale of activities is assumed; 4) for the overall assessment of the effectiveness of activities, financial and economic coefficients with weighting coefficients determined by experts are used. This makes the assessment subjective. The disadvantages of coefficient analysis of the activity of complex systems are investigated in [3].

The methods of the second group are based on the concept of efficiency limits. The efficiency of business entities is calculated considering the approximation of the values of performance indicators

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of an individual entity to the efficiency limit. There are parametric and nonparametric methods for approximating the efficiency limit. Parametric methods define the functional form of a production function, the parameters of which are determined by econometric methods. Nonparametric methods perform a piecewise linear approximation of the effective limit by mathematical programming methods. Parametric methods can analyse the influence of factors of production on one of the results of production. Nonparametric methods can analyse the influence of several factors of production simultaneously on the results of production. The advantage of parametric methods is the consideration of observation errors in the initial data, while nonparametric methods are flexible in approximation. The efficiency estimated by parametric methods is sensitive to changes in the input and output values, and the composition of the studied population. Efficiency estimated by nonparametric methods is sensitive to changes in the input, output values, and composition of only fully efficient business entities. Nonparametric methods determine the relative efficiency, that is, the efficiency of an individual economic entity relative to others. Parametric methods evaluate and analyse the effectiveness of an individual business entity.

The study of the efficiency of crop production by agricultural enterprises of Ukraine will be carried out using the parametric method, which is based on determining the functional form of the Cobb-Douglas production function. The Cobb-Douglas function determines the relationship between the volume of production and the costs of two types of resources involved in production (labour and natural resources, fixed assets, investments, etc.). The use of this function allows predicting and planning production volumes, identifying the contribution of each resource to the overall growth of production, evaluating the efficiency of production and resource use, determining the possibility of their interchangeability, identifying the impact of the scientific and technical process on production processes, the type of economic growth (extensive or intensive), and assessing the scale of production and its impact on production efficiency. The Cobb-Douglas production function can be used to analyse and predict the production activities of both individual business entities and economic associations, sectors of the economy. Thus, the use of the Cobb-Douglas function for planning crop production by agricultural enterprises in Ukraine is relevant.

There are many scientific studies devoted to the theory and practice of applying the Cobb-Douglas production function to assess the processes of production activities of various business entities. Among the foreign researchers who have been engaged in these studies in recent years: V. Akhundov [4], F. Cysneiros, V. Leiva, S. Liu [5], R. Smirnov R., K. Wang, Z. Wang [6], M. Hadush [7], Y. Yang, L. Yang, Shou Yi Liu [8], P. Ghoshal, B. Goswami [9] et al. The study by V. Akhundov [4] deals with the problem of predicting the regional final product using the Cobb-Douglas fuzzy model. F. Cysneiros, V. Leiva, S. Liu [5] proposed a methodology for modelling the Cobb-Douglas function with stochastic constraints. In the study by R. Smirnov, K. Wang, Z. Wang [6] the Cobb-Douglas production function is constructed considering the exponential growth of production. In the study by M. Hadush [7] using the Cobb-Douglas function, the relationship between agricultural production, agricultural labour costs, and resource scarcity are studied. In the study by Y. Yang, L. Yang, Shou Yi Liu [8], the Cobb-Douglas function is constructed to analyse the impact of capital and labour on the economic growth of agriculture in one of the provinces of China. P. Ghoshal, B. Goswami [9] proposed to use the Cobb-Douglas production function to measure the efficiency of agriculture in individual regions of India, and identified factors affecting the efficiency and inefficiency of agricultural production. Among the Ukrainian studies devoted to the application of the Cobb-Douglas production function, the following works can be mentioned: B. Hrabovetskyi [10-12], O. Fomina, N. Hoholieva, Yu. Labuzova [13], Yu. Kharazishvili, V. Liashenko [14], S. Shumska [15; 16], V. Yankovyi [17] et al. B. Hrabovetskyi [10] defined the essence and areas of using production functions in production management, highlighted the need to consider the features of the industry under study when forming the composition of factors of production function, and revealed the features of using the Cobb-Douglas function for the sugar beet industry. In the studies [11; 12], B. Hrabovetskyi investigated the use of production functions in the production management of certain branches of the food industry of Ukraine, namely, fat and oil, confectionery, and sugar beet industries. A. Fomina, N. Hoholieva, Yu. Labuzova [13] constructed the production function of the Ukrainian economy and gave recommendations for improving the economic growth policy. Yu. Kharazishvili and V. Liashenko [14] developed a modification of the Cobb-Douglas function to assess the innovative contribution to economic growth, in particular, in the old industrial regions of Ukraine. S. Shumska [15] presented the main steps of constructing the Cobb-Douglas production function for the Ukrainian economy. V. Yankovyi [17] developed theoretical and methodological aspects of modelling the growth factors of an enterprise based on production functions.

Analysis of publications devoted to the theory and methodology of production functions indicates that in recent years, researchers have paid attention to the use of various modifications of the Cobb-Douglas function, including applied research on the use of the Cobb-Douglas function at the macro and micro levels of the economy. The issues of applying the Cobb-Douglas production function for forecasting, evaluating agricultural production processes in Ukraine, and analysing the causes of inefficient activities have not been fully investigated.

The purpose of the study is to determine the specifics of applying the Cobb-Douglas production function to improve the planning of crop production by agricultural enterprises of Ukraine.

▸ Materials and Methods

Economic statistics data for building an econometric model of crop production planning by agricultural enterprises of Ukraine in the form of the Cobb-Douglas production function is obtained from the official website of the State Statistics Service of Ukraine and the statistical collection "Agriculture of Ukraine" (2021). Including statistical data of the index of crop production in agricultural enterprises for the period 1996-2009 obtained from the website of the State Statistics Service of Ukraine from the section "Indices of agricultural products (1991-2009)", and for the period 2010-2020 – from the statistical collection "Agriculture of Ukraine" (2021). Statistical data of the index of agricultural land areas are calculated based on statistical data of agricultural land areas in crop production, which are obtained from the website of the State Statistics Service of Ukraine from the section "Crop production (1991-2020)". Statistical data of the index of labour productivity in agricultural enterprises in crop production was obtained from the website of the State Statistics Service of Ukraine from the sections "Indices of agricultural products (1990-2009)" and "Indices of agricultural products (2010-2020)".

The study was conducted using a generalised heuristic method, which included the following stages:

1. Preliminary statement of the research task: 1) determination of the object of modelling (crop production by agricultural enterprises of Ukraine); 2) modelling goals (analysis, forecast, planning of crop production by agricultural enterprises of Ukraine); 3) modelling apparatus (parametric classes of functions);

2. Investigation and analysis of the research task: 1) collection and study of information on the research topic; 2) assessment of the degree of relevance of the study;

3. Clarification and detailing of the problem statement: 1) selection of the type of parametric function (production function) and its modifications (Cobb-Douglas function); 2) determination of input and output parameters of the production function (development of the composition of factors of the production function considering the characteristics of the crop production industry, checking the tightness of the relationship between input and output factors, selection of the most important factors);

4. Implementation of the research task (determination of unknown model parameters, analysis of the quality of the model and its predictive properties);

5. Analysis and evaluation of the solutions found.

The study also applied the following methods: 1) statistical research (for collecting, processing, and analysing the initial data, which included assessing the tightness of the relationship between the initial data based on calculated paired and partial correlation coefficients); 2) economic and mathematical modelling (for choosing the form of dependence of the Cobb-Douglas production function); 3) econometric modelling (for building the Cobb-Douglas production function based on statistical data); 4) correlation and regression analysis (for analysing the quality of the constructed model and its forecast properties); 5) special (parametric method for approximating the efficiency limit, least squares method for determining unknown parameters of the Cobb-Douglas function); 6) computer (using the MS Excel add-in "**Search for a solution**" to determine the parameters of the Cobb-Douglas function and standard MS Excel functions for analysing its quality and predictive properties); 7) abstract and logical (for drawing conclusions).

▸ Results and Discussion

In 1928, the American mathematician, economist C. Cobb and the American economist P. Douglas proposed the following mathematical model of the dependence of output volumes on labour and capital [18] (1):

$$Q = AL^{\alpha}K^{1-\alpha} \quad (1)$$

where Q – production volume; L – labour costs; K – capital expenditures; A, α – positive coefficients that characterise the production technology.

Currently, the improvement of the Cobb-Douglas production function is carried out in the following areas [19]:

– use of relative values, a wide range of variables that reflect different types of factors of production;

– accounting for conditions of uncertainty and turbulence of the external environment;

– accounting for the delay in the results of production activities in relation to the time of influence of factors due to the use of lag variables;

– consideration of the impact of technological progress, innovative factors of economic growth, and labour quality;

– accounting for the impact of climate factors through the use of fictitious variables;

– application of correlation and variance analysis to determine the degree of influence of factors on production results, etc.

Improvement of crop production planning by agricultural enterprises of Ukraine will be carried out through the development of an econometric model in the form of the Cobb-Douglas production function (2):

$$Y = b_0 K^{b_1} L^{b_2} \quad (2)$$

where b_0 – proportionality or scale factor; Y – index of crop production in agricultural enterprises; K – index of agricultural land areas; L – index of labour productivity in agricultural enterprises in the field of crop production; α, β – coefficients of elasticity for capital and labour, respectively.

The study approximates the data in Table 1 by equation (2).

Table 1. Initial data for constructing the Cobb-Douglas function for planning crop production by agricultural enterprises of Ukraine (1996-2020)

Year	Index of crop production in agricultural enterprises (y_i) [20; 21]	Index of agricultural land area (K_i) [22]	Index of labour productivity in agricultural enterprises, crop production (L_i) [23; 24]
1996	81.2	95.27	86.4
1997	110.7	107.8	111.2
1998	78.1	93.7	82.3
1999	91.5	99.28	92.4
2000	105.4	102.42	112.3*
2001	122.4	107.56	127.7*
2002	94.1	100.3	117.8*
2003	70.3	91	92.4
2004	155.5	112.01	176.4
2005	96.6	98.38	110.4
2006	106.1	99.22	113.7
2007	89.9	99.99	96.8
2008	148.8	104.44	150.2
2009	90.2	100.23	97.4
2010	94.6	99.01	97
2011	134.9	104.03	125.3
2012	91.0	100.45	92.4
2013	123.6	101.61	129.5
2014	103.9	94.82	108.2
2015	94.5	98.26	95.8
2016	112.4	103.16	121.8
2017	96.2	100.96	96.7
2018	113.6	101.09	115.8
2019	102.5	101.02	106
2020	85.8	102.66	89.4

Note: * – Index of labour productivity in agricultural enterprises
Source: State Statistics Service of Ukraine [20-24]

The unknown parameters of the function (2) are calculated using the least-squares method from the following equation (3):

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}(y_i - Y_i)^2 = \sum_{i=1}^{n}(y_i - b_0 \cdot K_i^{b_1} \cdot L_i^{b_2})^2 \to min \qquad (3)$$

where y_i – actual indices of crop production in agricultural enterprises, n – the number of observations, n=25.

It is possible to minimise the function (3) using the MS Excel add-in "**Search for a solution**", the application of which does not require the linearisation of the function, which leads to a shift in parameter estimates. As a result, the following coefficient values are obtained: b_o=0.06, b_1=0.77, b_2=0.81.

Therefore, the regression equation (3) will have the following form (4):

$$Y = 0.06K^{0.77}L^{0.81} \qquad (4)$$

Table 2 shows indicators for evaluating the overall quality of the model (4).

Table 2. Indicators for evaluating the overall quality of the Cobb-Douglas function for planning crop production by agricultural enterprises of Ukraine

Indicators	Equation	Calculated value
Coefficient of determination	$R^2 = 1 - \frac{\sum(y_i - Y_i)^2}{\sum(y_i - \bar{y})^2}$, $\bar{y} = \frac{1}{n}\sum y_i$	0.89
Multiple coefficient of determination	$R = \sqrt{R^2}$	0.95
Adjusted coefficient of determination	$\overline{R^2} = 1 - (1 - R^2)\frac{n-1}{n-m-1}$ m – the number of explanatory variables, m=2.	0.88
Standard approximation error	$S = \sqrt{\frac{\sum e_i^2}{n-m-1}}$	7.04

Source: calculated by the author

Analysis of the calculated indicators for evaluating the overall quality of the model indicates a high quality of the model (4). The calculated value of the coefficient of determination R^2=0.89 indicates that 89% of the statistics are approximated by the constructed model. Next, the study analyses the statistical significance of the calculated coefficient of determination. First, the value of F – Fischer's criterion is calculated by equation (5):

$$F = \frac{R^2}{1-R^2}\frac{n-m-1}{m} \quad (5)$$

Thus, F=92.94. Using the Fischer critical point tables, the critical value F_{cr} $(\alpha; m; n-m-1)=F_{cr}$ $(0.05; 2; 22)=3.44$ is found. Since the calculated value – the coefficient of determination is statistically significant and correctly reflects the quality of the model.

The forecast properties of the model (4) are estimated by the indicators of the absolute average percentage error of the forecast and the average approximation error. Table 3 shows indicators for evaluating the predictive properties of the model (4).

Table 3. Indicators for evaluating the forecast properties of the Cobb-Douglas function for planning crop production by agricultural enterprises of Ukraine

Indicators	Equation	Calculated value
Average forecast error	$ME = \frac{1}{n}\sum_{i=1}^{n} e_i$	0.04
Average absolute percentage error	$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left\lvert\frac{e_i}{y_i}\right\rvert 100\%$	4.5%
Average percentage error	$MPE = \frac{1}{n}\sum_{i=1}^{n}\frac{e_i}{y_i}100\%$	-0.5%
Average approximation error	$V = \frac{S}{\bar{y}}100\%$	6.8%

Source: calculated by the author

Next, the study analyses the predictive properties of the constructed model (4) based on the indicators from Table 3. The average forecast error characterises the degree of forecast bias [25]. If the forecast values are generally overestimated, then ME<0, if underestimated – ME>0. The calculated value of the average forecast error of the model (14) ME≈0, which indicates that there is no bias in the forecast. It is considered [25] that if the calculated value $MAPE$ is less than 10%, then the model provides high forecast accuracy. If the value changes in the range from 10 to 20%, then the model provides good accuracy, and in the range from 20 to 50% – satisfactory accuracy. Value $MAPE$ more than 50% indicates unsatisfactory forecast accuracy. Based on calculations: $MAPE$≈4.5%. Since the calculated value $MAPE$ is less than 10%, then the forecast for the constructed model will be highly accurate.

The average percentage error is an indicator of forecast bias and should not exceed 5% [25]. Based on calculation, MPE=-0.5%, i.e., there is no offset. The value of the average approximation error determines a relative forecast error. Based on the calculation results: V≈6.8%, in other words, the model provides high forecast accuracy. Therefore, the predictive properties of model (4) are high. The constructed Cobb-Douglas production function allows forecasting the volume of crop production by agricultural enterprises of Ukraine and analysing the efficiency of using agricultural land and labour resources. Next, the study determines the average labour productivity and average fund return using equations (6-7):

$$AP_L = \frac{Y}{L} = \frac{0.06K^{0.77}L^{0.81}}{L} = 0.06K^{0.77}L^{-0.19} \quad (6)$$

$$AP_K = \frac{Y}{K} = \frac{0.06K^{0.77}L^{0.81}}{K} = 0.06K^{-0.23}L^{0.81} \quad (7)$$

Average values of coefficients calculated for the period 1996-2020 AP_L, AP_K are equal to 0.86 and 0.93, respectively. This means that on average, 1% of the labour productivity index in agricultural enterprises accounts for 0.86% of the crop production index and 1% of the agricultural land area index accounts for 0.93% of the crop production index.

Next, the study determines the marginal productivity and marginal fund return using equations (8-9):

$$MP_L = \frac{\partial Y}{\partial L} = (0.81)(0.06)K^{0.77}L^{-0.19} = 0.049K^{0.77}L^{-0.19} \quad (8)$$

$$MP_K = \frac{\partial Y}{\partial K} = (0.77)(0.06)K^{-0.23}L^{0.81} = 0.046K^{-0.23}L^{0.81} \quad (9)$$

Average values of coefficients calculated for the period 1996-2020 MP_L, MP_K are equal to 0.70 and 0.71, respectively. This means that on average, 1% increase in the labour productivity index accounts for 0.70% of the increase in the crop production index; and 1% increase in the agricultural land area index accounts for 0.71% of the increase in the crop production index.

When modelling, it is important to find out how sensitive the value of the dependent variable Y is before changing factors K and L. It is known that the parameters b_1 and b_2 represent the elasticity of output by capital (b_1) and for labour (b_2). It is considered that an increase in capital expenditures by 1% leads to an increase in output by b_1%, and an increase in labour costs by 1% – to an increase in output by b_2%.

Calculated value of elasticity of crop production output by area of agricultural land $E_{Y.K.}=b_1=0.77$ means that if the agricultural area index is increased by 1%, the crop production index will increase by 0.77%. Elasticity of crop production output in terms of labour productivity $E_{Y.L.}=b_2=0.81$ means that if the labour productivity index is increased by 1%, the crop production index will increase by 0.81%. Since the elasticity of crop production output in terms of productivity is greater than the elasticity of output in terms of sown areas, agricultural enterprises use labour more intensively than capital. Since the values of the elasticity coefficients $E_{Y.K.}$, $E_{Y.L.}$ are less than one, – the output of crop production is relatively inelastic in terms of labour and crop area.

It is known that if $b_1+b_2>1$, then equation (4) has an increasing effect on the scale of production. This means that when increasing K and L in some proportion, Y increases in a larger proportion. If $b_1+b_2=1$, – there is a constant effect on the scale of production. This means that Y increases in the same proportion as K and L. If the condition $b_1+b_2<1$ is met, there is a negative effect. This means that Y increases in a smaller proportion than K and L. Therefore, using the Cobb-Douglas function, it is possible to estimate the degree of return on the scale of production. Next, the study calculates the return on the scale of production studied using the equation (10):

$$b_1 + b_2 = 0.77 + 0.81 = 1.58 > 1 \quad (10)$$

Thus, a growing return on scale is obtained. This means that the increase in crop production is carried out in a larger proportion than the increase in acreage and labour productivity.

Conclusions

In the study, the improvement of crop production planning by agricultural enterprises of Ukraine is carried out through the development of the Cobb-Douglas production function. The paper proposes a production function that describes the relationship between the index of crop production by agricultural enterprises of Ukraine and the indices of agricultural land areas and labour productivity. The parameters of the Cobb-Douglas production function are determined and the high quality of the constructed model is proved. To predict the constructed function, the predictive properties of the model are estimated and it is proved that the predictive properties of the model are high. To analyse the efficiency of using resources in the production process under study, the coefficients of average, marginal products of labour and capital are calculated. It is determined how much of the crop production index accounts for an average of 1% of the labour productivity index (0.86%) and 1% of the agricultural land area index (0.93%) during the study period. It is determined how much the average 1% increase in labour productivity indices and agricultural land areas accounts for the increase in the crop production index (0.70% and 0.71%, respectively). It is revealed how sensitive the value of crop production is to changes in crop areas and labour productivity. It is determined that an increase in the agricultural area index by 1% leads to an increase in the crop production index by 0.77%, and an increase in the labour productivity index by 1% leads to an increase in the crop production index by 0.81%. Thus, investment in labour resources and capital gives approximately the same effect in the growth of crop production. The presence of an increasing return on the scale of production is revealed, which indicates the feasibility of further expansion of the crop production industry.

The findings can be used in planning the crop production by agricultural enterprises, analysing the efficiency of production and resource use. The use of the developed model would allow making informed management decisions in conditions of uncertainty and turbulence of the external environment.

The prospect of further study is the analysis of priority areas of agricultural production in Ukraine using the method of production functions.

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Вдосконалення планування виробництва продукції рослинництва в Україні за допомогою виробничої функції Кобба-Дугласа

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▸ **Анотація.** Актуальність дослідження обумовлена необхідністю вдосконалення процесу планування виробництва продукції рослинництва сільськогосподарськими підприємствами України через побудову функціональної залежності між результатами виробничої діяльності підприємств та факторами, що обумовлюють ці результати. Мета статті – вдосконалити процес планування виробництва продукції рослинництва сільськогосподарськими підприємствами України через розробку економетричної моделі у вигляді виробничої функції Кобба-Дугласа. Для моделювання процесів виробництва продукції рослинництва в роботі запропоновано використовувати виробничу функцію Кобба-Дугласа, яка описує залежність індексу продукції рослинництва від індексів площ посівів сільськогосподарських угідь та продуктивності праці. На основі статистичної інформації за 1996–2020 рр., методами регресійного аналізу знайдені невідомі параметри виробничої функції Кобба-Дугласа. Доведено високу якість побудованої моделі. Доведено високі прогнозні властивості моделі. Для аналізу ефективності використання ресурсів при виробництві продукції рослинництва сільськогосподарськими підприємствами розраховані коефіцієнти середніх, граничних продуктів праці та капіталу. Досліджено чутливість величини випуску продукції рослинництва до зміни площ посівів та продуктивності праці. Визначено, що якщо індекс сільськогосподарських площ збільшити на 1 %, то індекс продукції рослинництва збільшиться на 0,77 %, а якщо індекс продуктивності праці збільшити на 1 %, то індекс продукції рослинництва збільшиться на 0,81 %. Виявлено зростаючу віддачу від масштабу витрачених ресурсів, що свідчить про збільшення обсягів виробництва при збільшенні обсягів ресурсів в галузі рослинництва

▸ **Ключові слова:** економетрична модель, оцінка якості моделі, прогноз, ефективність використання ресурсів, еластичність випуску продукції, ефект масштабу, сільськогосподарські підприємства