



**ГЛАВА 11. DOI:10.30888/978-617-7414-51-2.0-006**  
**THE LAGRANGE METHODS FOR OPTIMIZATION**  
**OF INTERACTION IN AQUATIC SYSTEMS**

**Introduction.**

It is evident that hydraulic one-dimension method based on the average flow velocity brings to the unsusceptible model for aquatic systems optimization. According to the widespread hydrodynamic Euler method we should study a velocity field connected with the local velocity concept under solving the Navier-Stokes equations (Atanov, Daugherty 1985). To the contrary, the Lagrange method deals with the individual water particles and it seems to be more natural approach. However, for unknown flow lines this method has not the advantage by mathematical difficulties. Nevertheless, we have the row of engineering problems characterized with the quite defined boundary and initial conditions regarding to the some particular elements of flow (Daily & Harleman 1971, Wirz & Smolderen 1978). For example, the boundary streams move along the flow formative lines and it is the important circumstance to use the Lagrange method for calculation of interactions between flow and streamlining bodies (Shandyba et al.1992, 1998, 1999). On the other hand, filtration moving along relief slope allows to solve the mass-transfer equations and predict the ecology consequences of chemical substances migration (including radionucleades) into ground water and the residue levels of the dangerous chemicals in soil (Shandyba et al. 1997). This simple and comprehensive method also can be used to improve the design and operation of recycling water treatment of manufacturing process systems (USSR Patent No 1761819). At the same time, there is growing technical concern about the available optimization procedures which take into account the easy-defined integral parameters of hydrodynamic, chemical or biology interactions in different aquatic systems. On this reason, an attempt is made up for an extension of this field.

**11.1. Pollution migration forecast for soil-geochemistry mapping**

The migration of the moving dangerous chemicals in soil-water systems represents significant risk to public health and environment. At the present time there is growing scientific concern about the available predicting procedures for environmental assessment of contaminated sites and chemical spills. After considering the various approaches and geodata that may be involved, the stagnate zones model was recognized. The key problem to be considered here deals with the surface concentration distribution, risk evaluation and allowable residue levels for chemicals. It is possible to make forecast and ecology monitoring based on the proposed mathematical model with tabulated migration parameters of the contaminants and soils. The considered method can complement experimental work on the contaminated sites and assist with soil-geochemistry mapping.

To identify this characteristics for interrupted migration regime we must find the adequate experimental dependence of their relation on rain (snowmelting) intensity. The likeness of the experimental curves which correspond to interrupted as well as uninterrupted regimes produces the reasons for achievement of the predictive aims of



the suggested model under native weather conditions.

The two-dimension experiment was carried on the thin-layer model. A soil sample were placed in the box  $100 \times 60 \times 10$  cm. In order to measure the concentration distribution into layer it was first necessary to apply the conductometer method. The result for relational conductivity shows that the pollution redistribution caused by precipitation depends strongly upon the box slope, soil structure and individual distance from watershed. Similar experimental results were gained by Kremlenkova, Bell, Patrzalek, Hulten, Arias et al. [1, 2, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

A very important application of this investigation was to predict the pollution area as the ecological consequence of the chemical substances migration in soil-water systems. The results of modelling under real landscape (Fig.3) and weather conditions are presented as the residue relational pollution for the average concentration fields. This soil-geochemistry mapping demonstrates great changes of the concentration isolines which are due to relief, geotechnical properties of soil and precipitation intensity.

On the other hand, Jessberger, Bresler et al., Kremlenkova & Patrzalek [4, 5, 6, 9] show the experimental results to the characterisation of waste disposal sites. Various natural soil, plants and landscape geochemical agents have been considered as accumulating for the technogenic soil pollution. The content of dangerous elements depends not only the technogenic sources intensity, but also on soil texture, redox conditions, relief slope and location of soil in the system of elementary landscapes. Under real environmental management it is possible to make a risk assessment, based on the integer loss function calculated for limiting contaminant at the control border

$$f = \frac{\int_L \int_{\tau} \psi q C x dl d\tau}{M_0 S} \quad (1)$$

where  $\psi$  - hydraulic transfer parameter;  $q$  - rain (snowmelting) intensity,  $m^3 / HaS$ ;  $C$  - actual concentration defined by the eqs.(11,12);  $x$  - length of a flow line from watershed to control border  $L$ ;  $dl$  - element of control border;  $d\tau$  - element of summary time of precipitation;  $M_0$  - initial contaminant content,  $Kg/Ha$ ;  $S$  - control area,  $Ha$ .

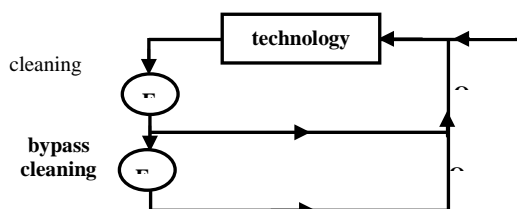
## 11.2 Differential recycling water supply system

The use of recycling water supply systems under non-blowing operation results in the contaminants accumulation and exceeding concentration limits for circulation water (Shandyba 1996, Ukraine Patent No 20947 A). Balance of accumulation and contaminant removing is provided by treatment plants (cleaners) which usually are special-purposed for several contaminants (Figure 1). However, it is necessary to remove all dangerous limited components from circulation water. Moreover, a different intensity of a different contaminants accumulation in circulation water must be compensated by means of an adequate intensity of removing these components from water on the treatment stages. To prevent exceeding of the allowable level of the contaminant concentration in circulation water the following condition should be provided:



$$C_i E_i Q + C_i E_{ip} Q_p = (r + g) C_{io} + M_i - G_i \quad (2)$$

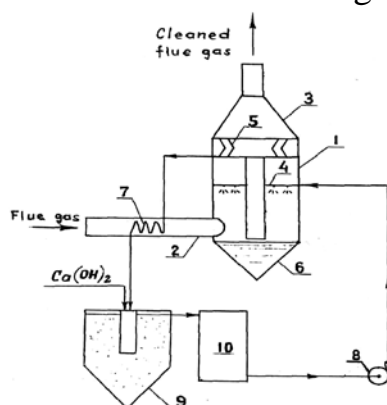
where  $C_i'$  - limit concentration of  $i$  contaminant in circulation water;  $C_i$  - actual concentration of  $i$  contaminant in circulation water;  $C_{io}$  - initial concentration of  $i$  contaminant in fresh water;  $E_i$  - removing efficiency on the cleaning plant;  $E_{ip}$  - removing efficiency of  $i$  contaminant on the bypass cleaning plant;  $Q$  - circulation water rate;  $Q_p$  - bypass water rate;  $M_i$ ;  $G_i$  - accumulation and loss intensity of  $i$  contaminant;  $(r + g)$  - evaporation and hydraulic transfer rate.



**Fig.1. Differential recycling water supply system**

To realize the suggested approach in practice we have proposed the two-staged waste heat utilization plant (figure 2). The operation is possible if the intensity of hardness accumulation in water does not exceed some level determined eq.(1) under efficiency corresponding to the heat output of flue gas flow (Ukraine Patent No 20947 A). Evidently, if the heat output is not sufficient for heating the whole circulation water rate, then it can be enough for heating bypass water. The sample of heat utilization plant made as a scrubber 1 with flue gas inlet 2 and outlet 3, sprayers 4, drop separator 5 settling cone 6. Besides the system includes heat exchanger 7, circulation pump 8, neutralizator 9 and stabilizator 10.

Under operation, the flue gas through the inlet 2 flows into the scrubber 1, heats the sprayed circulating water to 55-65°C and arises with the captured water drops to the drop separator 5. After the separation, a some part of the circulating water passes through the heat exchanger 7 to the neutralizator 9. The lime slurry neutralization of high temperature water brings to more hardness reducing and water stabilization.



**Fig. 2. Flue gas heat utilization plant**

### 11.3. Power interaction in water flow

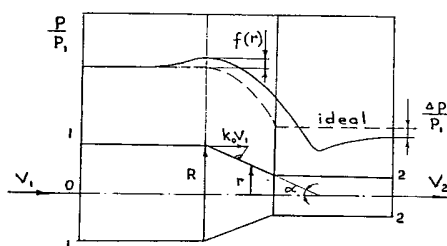
Theory of head-resistance under contraction. The conical contraction is the most widespread unit of many technical systems. For the inside problem of



Hydrodynamics it is the noticeable sample of power interaction between flow and streamlining surface (Daily & Harleman 1971, Shandyba 1992, 1999). In this consideration we shall be limited by the developed turbulence regime that allows to examine the influence of contraction geometry on pressure distribution, energy losses and drag resistance. It was found that the loss of pressure in axially symmetric conical contraction (figure 3) is connected with the excess pressure of viscous flow to ideal flow by the following equation:

$$\Delta p S_2 = 2\pi \int_r^R f(r) r dr \quad (3.1)$$

where  $\Delta p$  is loss of pressure,  $S_2$  is lesser cross-sectional area,  $r$ ,  $R$  are radii of lesser and greater cross sections,  $f(r)$  is excess pressure of viscous flow to ideal flow.



**Fig. 3. Excess pressure of viscous flow to Ideal flow**

To determinate this function  $f(r)$  we suppose the whole flow in the contraction as the complex of elementary streams where pressure and velocity are averaged on time according to the Reynolds-Boussinesq model. Taking into account the change of the flow structure in contraction, one must consider the two characteristic sections of flow: before and into contraction. The character of interacting each stream with the conical surface depends on its initial disposition in flow before contraction and the contraction geometry. At this point of view the boundary streams seem to be most important. Under the unseparated streamlining movement these have the quite defined ways like the contraction formative lines.

Using the impulse conservation equation the excess pressure can be found for the boundary streams. Thus, if a liquid particle with mass equal  $\rho$  has the impulse  $\rho(k_0 V_1)$  in cross-section 1-1 (where  $k_0$  is ratio of boundary stream velocity to average flow velocity before contraction), then its impulse will be equal  $\rho(k_0 V_1) \cos \alpha$  after interacting with the conical surface under attack angle  $\alpha$ . The corresponding excess pressure in the connection point of conical contraction will be defined from Bernoulli's equation:

$$p(R) = p_1 + \frac{\rho(k_0 V_1)^2}{2} - \frac{\rho(k_0 V_1)^2}{2} \cos^2 \alpha = \quad (3.2)$$

$$= p_1 + \frac{\rho(k_0 V_1)^2}{2} \sin^2 \alpha$$

where excess pressure function  $f(R) = \frac{\rho(k_0 V_1)^2}{2} \sin^2 \alpha$

The experimental data confirm the presence and proportionality of the excess pressure to  $\sin^2 \alpha$  function. It is important to note the increasing of excess pressure along the boundary streams on any head streamlining surface under contraction



of flow. This takes place because there is energy redistribution in contraction connected with increasing energy of the boundary streams accordingly decreasing energy of the inside streams. The corresponding excess pressure occurs due to the change of impulses of the inside streams. The value of pressure change may be found from the following arguments. First, the excess pressure of real flow to ideal flow is the result of the interaction between flow and inside surface of contraction. It is connected with the changes of velocities and, accordingly, liquid particles' impulses in the streams. Moreover, only a part of impulse's energy is consumed for increasing potential energy of the boundary streams. This increasing conforms to  $\sin^2 \alpha$  function.

From the impulse conservation we can see that interaction of the inside streams with the contraction surface will be analogous with the boundary streams' interaction under their turning. In other words, a ratio of excess potential  $f$  and kinetic  $\varphi$  energy is kept constantly on all inside surface of the conical contraction,

i.e., if  $\alpha = \text{const}$ ,

$$\frac{df(r)}{d\varphi(r)} = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \text{const}$$

(3.3)

Generally speaking, the distribution of the excess energy in the boundary streams depends on the initial impulses distribution in flow and the local angles of interaction with the contraction surface. The excess energy distribution can be expressed as the sum:

$$dE = df(r) + d\varphi(r) = dE \sin^2 \alpha + dE \cos^2 \alpha \quad (3.4)$$

Second, the velocity of considering streams will increase in accordance with reduction of cross-sections of the contraction as well as the excess pressure will increase proportionally to the contraction degree function  $s = R^2 / r^2$ . It is very essential that the summary increase of kinetic energy of the boundary streams consists of the ideal and impulse components. The impulse component is increased by energy reduction of the inside streams having the liquid particles with  $\rho k V_1$  impulses, where  $k$  function increases from  $k_0$  to  $k_{\max}$  for axis. At the same time the ideal component is increased by Bernoulli's equation and correlated to  $s^2$  function in accordance with the continuous equation. On the same reason, the summary increase of kinetic energy of the boundary streams also is correlated to this contraction degree function.

Evidently, the impulse kinetic component  $d\varphi(r)$  is changed as the difference in analogous way:

$$d\varphi(r) = \frac{\rho(kV_1)^2}{2} \cos^2 \alpha d(s^2)$$

(3.5)

Therefore, from eq.(4) the excess pressure will depend on this function too:

$$df(r) = \frac{\rho(kV_1)^2}{2} \sin^2 \alpha d(s^2) \quad (3.6)$$

As integral, the excess pressure distribution on inside surface of contraction is:

$$f(r) = \frac{\rho V_1^2}{2} \int_1^s k^2 \sin^2 \alpha d(s^2) + f(R) \quad (3.7)$$

Then the shape component of head-resistance force can be defined as:



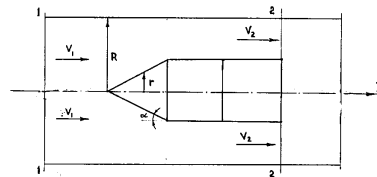


$$\Delta p S_2 = 2\pi \int_0^R \left[ \frac{\rho V_1^2}{2} \int_1^k \sin^2 \alpha d(s^2) + f(R) \right] r dr \quad (3.8)$$

**Axially symmetric model.** For a passage to the external aerodynamic problem we shall consider the peculiar ring contraction at the head of streamlining pontoon (figure 4). In this case we have a cylindrical long body with conical head situated in tube. Obviously, the hydraulic losses coefficient of the whole body may be presented as the sum:

$$\xi_x = \xi_h + \xi_f + \xi_s \quad (3.9)$$

where  $\xi_h$ ,  $\xi_f$ ,  $\xi_s$  are the hydraulic loss coefficients of the ring contraction, pontoon surface friction and Borda's sudden expansion after stern.



**Fig. 4. Ring contraction at the conical head of pontoon**

Assuming  $\alpha = \text{const}$ ,  $k = 1$  after integrating and transformation (8) we obtain:

$$\Delta p = \xi_h \frac{\rho V_1^2}{2} = (1/n^2 - 1/n) \sin^2 \alpha \frac{\rho V_1^2}{2} \quad (3.10)$$

where  $n = 1 - r^2/R^2$ .

From Borda's formula :

$$\Delta p = \xi_s \frac{\rho V_1^2}{2} = \frac{(1-n)^2}{n^2} \frac{\rho V_1^2}{2} \quad (3.11)$$

The share of these components can be calculated on the analogy of pipelines. As result, the total hydrodynamic resistance of pontoon under disregarding friction is:

$$F_x = \frac{\pi r^2}{n^2} \left[ (1-n) \sin^2 \alpha + (1-n)^2 \right] \frac{\rho V^2}{2} \quad (3.12)$$

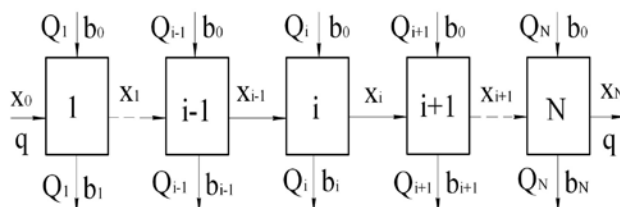
where contraction degree  $n = 0.695$  for free flow.

It is important to note that there is a possibility to optimize the power interaction by improving shape of streamlining bodies.

#### 11.4. Rational water consumption under multistage washing

The multistage washing with differential distribution of water rate on the stages is an efficient system for water-saving [15], [16], [17]. The typical example of this process refers to washing of dispersive materials in soil. The cascade method developed in some works [1-3] utilizes the matter balance and kinetic equations under unequal efficiency and mass-transfer parameters of every stages (Figure 5).

Washing with differential water distribution on stages. Assuming that contaminated stagnate zones volume are identical  $q_i \neq \text{idem}$  on the every stage but the mass-transfer Fourier numbers are unequal  $Fo_i = A_i \tau_i \neq \text{idem}$  we can find the optimal water distribution for maximum efficiency [3]. In other words, it is aimed to define the



**Fig. 5. Schema of multistage washing with differential water distribution on stages**

fresh water input for every stage to deliver the minimum relation  $\frac{x_N}{x_0} \Rightarrow \min$  under limited water consumption

$$\bar{Q}_N = \sum_{i=1}^N Q_i = \text{const} \quad (4.1)$$

and efficiency of washing process depends in a complex way on the mass-transfer parameters and water rate factors as follows:

$$E_i = \frac{C'_0 - C'_i}{C'_0 - C_0} = \frac{1 - \exp[-k_i t_i (1 + Q_i^{-1})]}{1 + Q_i^{-1}} \quad (4.2)$$

where  $C'_0$ ,  $C'_i$  - are initial and current average concentration of dispersion material in soil;  $C_0$  - is the initial average concentration of dispersion material in soil;  $k_i$  - is the volumetric coefficient of the mass-transfer;

$t_i$  - is the time of contact between dispersion material and water;  $Q_i$  - is the ratio of washing water rate to moisture zones rate of dispersion material  $q_i$

Simplifying for clean water  $C_0 \ll C'_i$  gives efficiency

$$E_i = 1 - \frac{C'_i}{C'_{i-1}} \quad (4.3)$$

After transformation we obtain

$$C'_N = C'_0 \prod_{i=1}^N (1 - E_i) \quad (4.4)$$

Then the multistage washing degree takes the form

$$\bar{C}_N = \frac{C'_N}{C'_0} = \prod_{i=1}^N (1 - E_i) \quad (4.5)$$

For minimizing the foregoing function the Lagrange system can be utilized

$$\frac{\partial}{\partial Q_i} (\bar{C}_N + \lambda \bar{Q}_N) = 0 \quad (4.6)$$

where  $\lambda$  is the Lagrange indefinite multiplier.

This system (6) includes  $N+1$  unknowns and may be solved with equation (1).

The solution of the explicit system results in the auxiliary function

$$\phi_i = \frac{\bar{C}_N}{1 - E_i} \frac{\partial E_i}{\partial Q_i} = \text{idem} \quad (4.7)$$

Thus, this result points for additive information on the relationship between the technological parameters of the mass-transfer in order to reach more rational clean water use.

### Concluding remarks.

The Lagrange methods are the good means for calculation of the optimal hydraulic and mass-transfer regimes as well as for improving process design. Considering the effect of differential intensity of mass-transfer as the specific



function of technological parameters on every stage gives some information to get the minimum water consumption. Multistage leaching experience has the evident arguments for the possible usage of limited water rate. The specimen results shown in the present report concern mainly water flows but may be used for optimization procedures in air protection plant. The effective management of hydraulic regimes, through improved system operation or new technology, leads to the more efficient use and conservation of resources and energy. It is important to note that there is a possibility to optimize the power interaction by improving shape of streamlining bodies.